TRANSFORMATION OF CHANNEL FORM AT A RIVER CONFLUENCE

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Natural sight of Bainganga River and confluence of its tributaries

ABSTRACT
Transformation of channel at a River confluence is deduced by replacing the hydraulic geometry relationships into the continuity equation of flow. Changes in the hydraulic geometry variable G can be described by: $G_0^x = G_1^x + G_2^x$ where the subscripts 0. 1 and 2 denote the receiving stream, major, and minor tributaries. The exponent may be a system-wide parameter$(\bar{x})$ or may be computed for individual junctions or). (x) In the first case, $\overline{x}$ is expected to be equal to the reciprocal of the exponent of the hydraulic geometry relationship between G and water discharge. In the latter case, x may be found by solving the above equation iteratively or from two nomographs presented in this paper. My own measurements of widths and slopes in a miniature drainage network, I will show that, although average morphometric changes at a junction may be adequately described by x, the behavior of individual junctions is highly variable reflecting important variations in x. The model may also be used to estimate the hydraulic geometry exponent which is assumed to be equal to the median of the x values. This application of the model is particularly useful in cases where water discharge is unknown.

Introduction

Few investigations have dealt with changes in channel geometry at a river junction. In geomorphology, I know of two simple models that describe such discrete changes. First, Miller (1958) proposed that changes in channel width (W) are such that

$$W_0 = \mathrm{p}(W_1 + W_2) \tag{1}$$

where 0, 1, and 2 denote the receiving stream, major, and minor tributaries meeting at the junction. The value of p usuallylies between 0.5 and 1.0. and Miller reported an average of 0.66 for high mountain streams. Park (1975) obtained an average p of 0.83. Miller also applied this approach to describe changes in channel depth, in cross-sectional area, in channel slope, and in particle size.

Richards (1980) pointed out several shortcomings in Miller's approach. Equation (1) only applies when the tributaries have similar widths, and asymmetrical branching implies that p is very large. In several cases, Miller's equation predicted a reduction in channel width below a junction. Furthermore, the value of p was a function of branching symmetry. Richards suggested an alternative model based upon the relationship between channel width and link magnitude(N), first proposed by Woldenberg (1972. p. 12)

$$W = wN^k \tag{2}$$

where w is the average width of streams with magnitude 1. The correlation coefficients for this relationship are very high. Woldenberg (1972) found that for 4 out of 5 tidal networks $r \geqslant 0.98$ Richards (1980) reported that $r = 0.97$ for the contour crenulated network and 0.92 for the blue line network of the River Baingagan (England). Equation (2) is viewed as a continuous relationship; Richards wished to estimate discontinuous changes at a junction. Accordingly, he proposed a width ratio that can be expressed as a ratio of link magnitudes (Richards 1980)

$$\frac{W_0}{W_1} = \left(\frac{N_0}{N_1}\right)^k. \tag{3}$$

This model improves upon Miller's equation (I) because it can be applied to symmetrical as well as to asymmetrical branching, and it always predicts an increase in channel width downstream of a junction. The evidence presented by Richards (1980) suggests that the magnitude ratio is a poor predictor of the width ratio, however. He found that $r = 0.64$ for the contour crenulated network and $r = 0.29$ for the blue line network of the River Baingagan. Clearly the method of network delineation had an impact on the strength of the relationship. The discrepancy between observed and predicted width ratios is also explained by the fact that a correlation between two ratios is inherently weaker than a correlation between two single variables. Finally, I find that a critical disadvantage of the model is that it considers only the receiving stream and the major tributary. The width and magnitude of the minor tributary are completely ignored, and since the ratio of tributary sizes is variable, this obviously will introduce a large component

Sight of Bainganga and its tributaries confluence

of unexplained variation. A model of channel geometry changes at a junction should take all three streams into account at once.

In this paper, I will develop a general model to describe changes in channel geometry occurring at a junction. The model is deduced from the substitution of the hydraulic geometry relationships into the continuity equation of flow, and it deals with all three streams simultaneously. I then proceed to show how the model can be applied. Although I do not present a comprehensive empirical analysis, our results suggest that the model is adequate to describe average changes in channel form but does not account for the great variability in morphometric changes at individual junctions. One interesting aspect of the model is that it can be applied backwards to deduce hydraulic geometry exponents from the morphometric adjustments observed at individual junctions.

A MODEL FOR GEOMETRIC CHANGES AT A JUNCTION

At a river junction, the discharge $(Q)$of the receiving stream equals the sum of the discharges of its tributaries

$$Q_0 = Q_1 + Q_2. \tag{4}$$

Assuming the hydraulic geometry equations are adequately represented by power functions, as was originally stated by Leopold and Maddock (1953), then I have

$$Q = \left(\frac{G}{a}\right)^{1/a} \tag{5}$$

in which G is a hydraulic geometry dependent variable (e.g., depth, width, crosssectional area, slope, roughness, velocity). The constant u is the appropriate hydraulic geometry exponent and a is the ordinate when $Q = 1$ (Leopold et al. 1964, p. 244). By substituting equation (5) in the continuity equation (4), it follows that

$$\bar{G}_0^x = G_1^x + G_2^x \tag{6}$$

where I expect x to be equal to the reciprocal of the exponent for the downstream hydraulic geometry relationship between G and Q. Clearly, if x is found directly from the best fit power function then equation (6) will only estimate the average change in G for a sample of junctions.

The critical assumption in this deduction is that a power function (eq. 5) is used as the fundamental relationship between a hydraulic geometry variable and discharge. Richards (1976) has shown that downstream hydraulic geometry relationships vary according to the pool-riffle sequence and to the channel pattern. His results indicated that the coefficients (but not the exponents) of the hydraulic geometry power functions differ if I look only at pools or only at riffles. These effects are difficult to embed into a general model, and deviations from equation (6) may therefore be systematic. Another problem is the sensitivity of the hydraulic geometry exponents to the flood frequency chosen as a basis for the relationship. Changes in channel geometry are most likely to be also a function of discharge. Nonetheless, for any given frequency of discharge, for example bank full, the model should adequately describe the average changes in form.

The exponent x may be viewed as a system-wide parameter or as a parameter for one specific junction. In the first case, $\bar{x}$ is derived from the hydraulic geometry relationships, and it represents an average for the whole system. Thus a way of testing the model is to use the hydraulic geometry exponents to predict the channel variable of the receiving stream,

$$\bar{G}_0 = \left(G_1^{\bar{x}} + G_2^{\bar{x}}\right)^{1/\bar{x}} \tag{7}$$

and to compare the predicted $(\bar{G}_0)$with observed values (G_0).

At a specific junction, on the other hand, the exponent x would precisely describe the

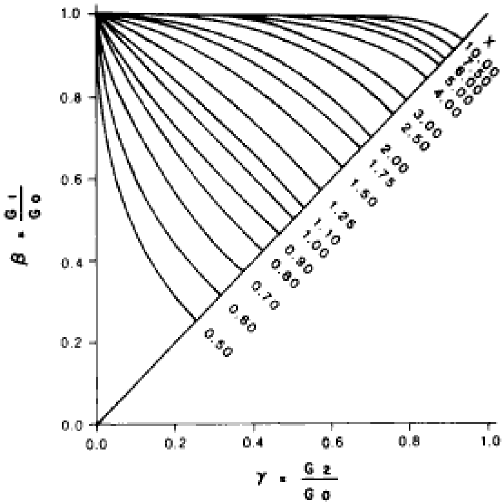


Figure: 1. The positive nomograph describing changes at a junction in channel width, depth, and usually velocity.

geometric changes. Given G_0, G_1, and G_2 at a junction, x is found by solving iteratively

$$\frac{G_1^x + G_2^x}{G_0^x} = 1. \tag{8}$$

The junction-specific exponents can be used to investigate changes in form with discharge and/or within a stream network. Although it is simple to solve equation (8) with a computer, the junction-specific exponent can also be determined from nomographs. These nomographs are particularly useful when only a few junctions are studied or as a means of depicting morphometric changes. Deriving the nomographs is a simple procedure, but it has to be done for two different cases.

The first case, where $x > 0$, is the positive nomograph. Here, $G_0 > G_1 \geqslant G_2$. this case would normally describe changes in width, depth, cross-sectional area, and velocity inasmuch as each usually increases downstream. Given the ratios

$$\gamma = \frac{G_2}{G_0} \text{ and } \beta = \frac{G_1}{G_0} \tag{9}$$

Equation (6) becomes

$$\beta^x + \gamma^x = 1. \tag{10}$$

Equation (10) is used to construct the nomograph shown in figure 1. Hence, given γ and β, an approximate value of x is easily found. For x to exist, $G_0 > G_1 \geqslant G_2$ and thus $\beta \geqslant \gamma$. I see on the nomograph that for high β, ($\beta > 0.9$) x can be highly variable, ranging from 0 to infinity. As β decreases so does x. and for all β smaller than 0.5. x is less than 1.0. Also, as γ increases, the range of x becomes restricted to higher values. Symmetrical branching implies that γ and β are nearly identical, and this corresponds to the diagonal of the graph. Asymmetrical branching ($\beta \gg \gamma$) is found in the upper left part of the graph. If γ and β are both small, major changes occur at the junction; that is. G_0 is much larger than G_1 and G_2. On the other hand, minor changes imply that γ and β are large. Accordingly, major changes in form at a junction are described by small values of x, and minor changes by large x. for large β the value of x is difficult to determine precisely, and the solution is impossible where β is equal to 1. For practical purposes, however, as x reaches 5.0, the left hand side of equation 10 is very close to 1.0.

The second case applies when $G_0 > G_1 \leqslant G_{2.}$ this relationship theoretically describes changes in slope, perhaps roughness, and occasionally velocity. In this case, I define γ' and β' with respect to the largest value (G_2) so that

$$\gamma' = \frac{G_1}{G_2} \text{ and } \beta' = \frac{G_0}{G_2}. \tag{11}$$

γ' and β' also lie between 0 and I, γ' being always greater than or equal to β' Equation (6) thus becomes

$$\beta'^x - \gamma'^x = 1. \tag{12}$$

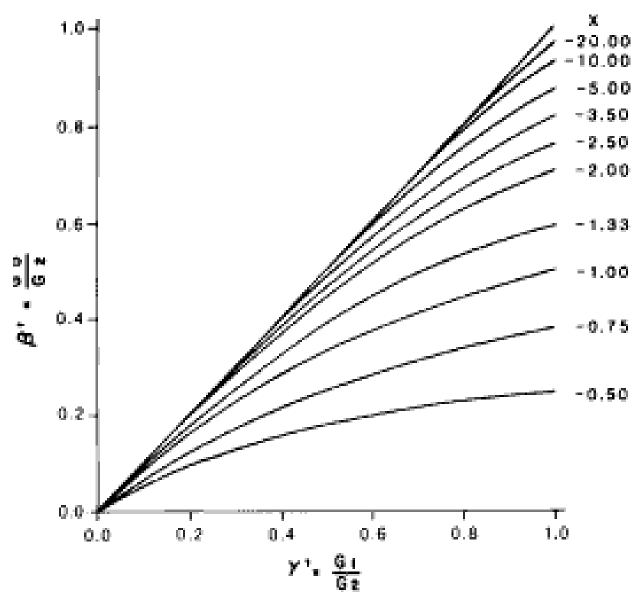


Figure: 2. The negative nomograph describing changes at a junction in channel slope, roughness, and perhaps velocity.

To solve this equation x has to be negative; this applies, for instance, to slope and roughness because they decrease as discharge increases downstream. The negative nomograph derived from equation (12) is plotted in figure 2. In contrast to the positive nomograph, the range of x here is not affected as γ' increases. In the positive nomograph small absolute values of x describe major morphometric changes at a junction. In the negative nomograph large absolute values of $x(x \leqslant 0)$ imply minor changes, that is, a low rate of change of G in a downstream direction. From the nomograph I immediately see that when γ' and β'are equal

$(\, i, e,$ when $G_0 = G_1\,), x$ is indeterminate. Here again one could set a minimum value for x which adequately describes slow changes in slope downstream. For example,$x = -1,$. Implies that the exponent of the slope-discharge relationship is equal to -0-1.

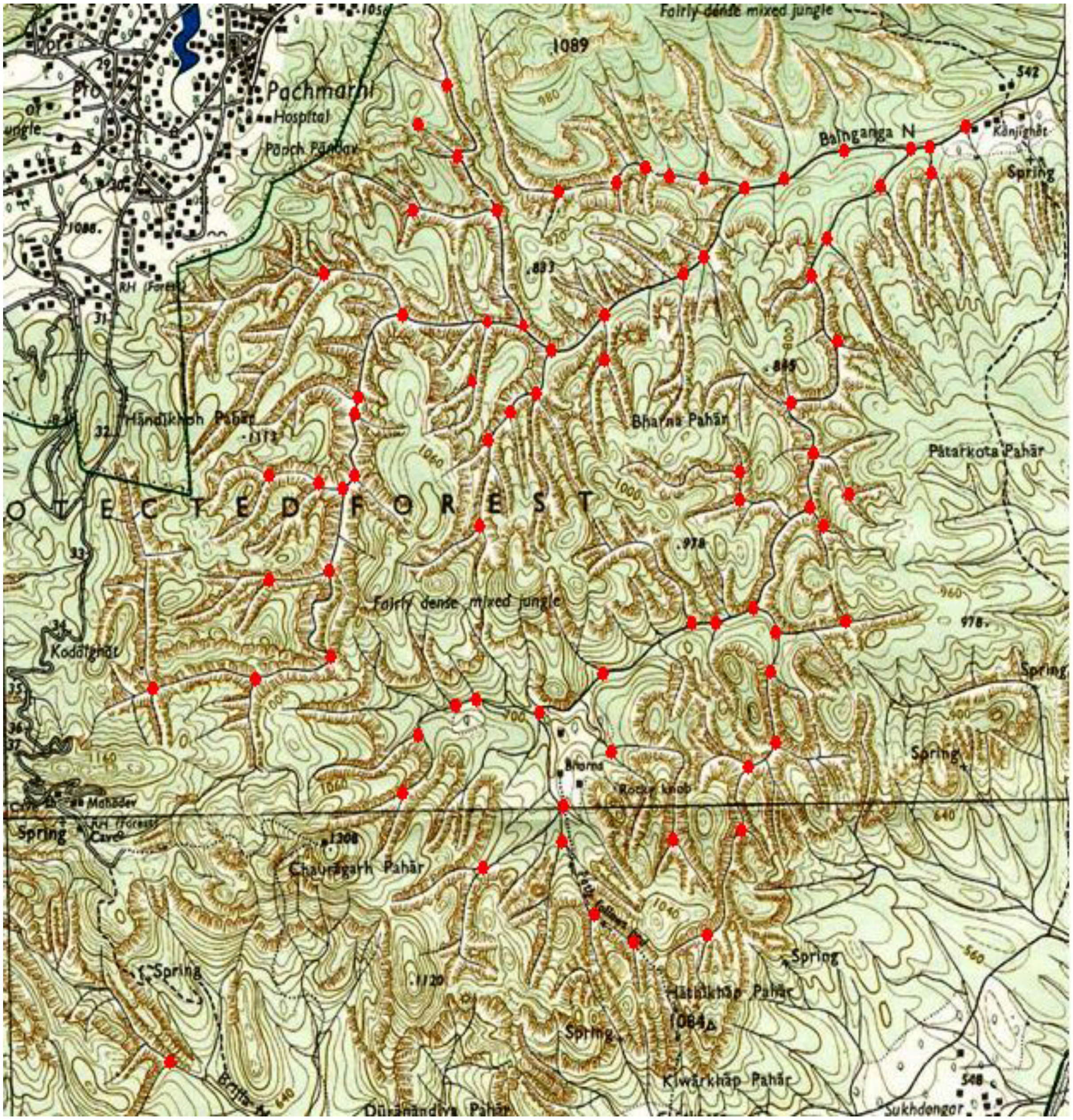


Figure: 3. River Bainganga drainage network(study area). Survey of India toposheet 55J7

APPLICATION OF THE MODEL

The simple model outlined in this paper has to be correct if the continuity equation of flow at a junction and the power function hydraulic geometry relationships are true. Changes in channel form at junctions can be estimated by the reciprocal of the hydraulic geometry exponent. To illustrate, I can compare the observed $G_0$with $\bar{G}_0$ calculated from equation (7), where x is found from the hydraulic geometry relationship between $G$and Q. I have measured channel widths at 10 junctions of the River Bainganga (figure :3) of Pachmarhis (North Satpura). The exponent (b) of the downstream hydraulic geometry relationships between W and $Q.$ I found that b is .33 for riffles and .35 for pools. Since the exponents were statistically *(* $p = .05$) undistinguished-able, I will use here a value of .34, and x is therefore equal to 2.94. Predicted widths are given by

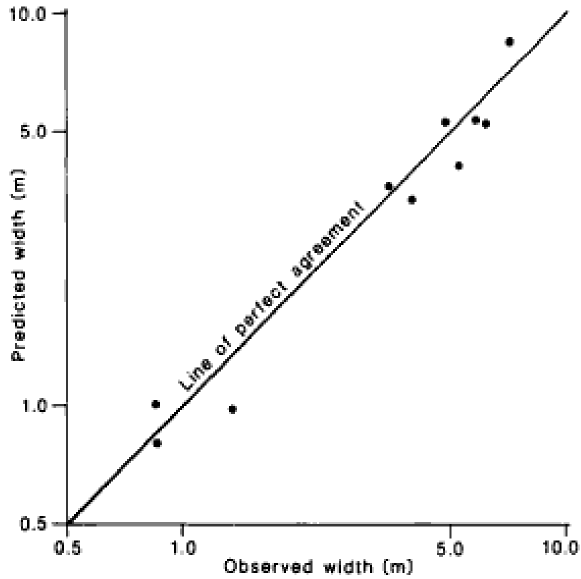


Figure: 4. Comparison of expected and observed widths for the River Bainganga of Pachmarhis (India), based on hydraulic geometry exponent, b = .34, $\bar{x}$ = 1/b= 2.94.

$$\bar{W}_0 = (W_1^{2.94} + W_2^{2.94})^{0.34} \tag{13}$$

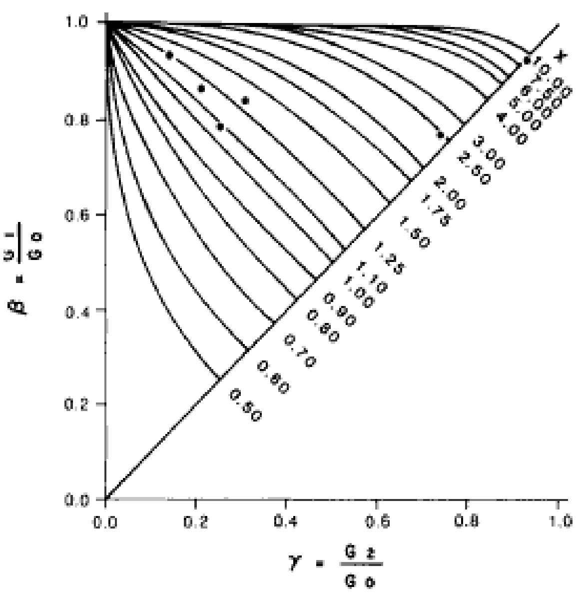


Figure: 5. Application of the positive nomograph to channel width for the river Bainganga

As illustrated in figure 4. Expected and observed widths are in very good agreement.

The model may also be applied to estimate x values and $\bar{x}$ and thus to determine the hydraulic geometry exponents of individual junctions and for the stream as a whole. The width data for the River

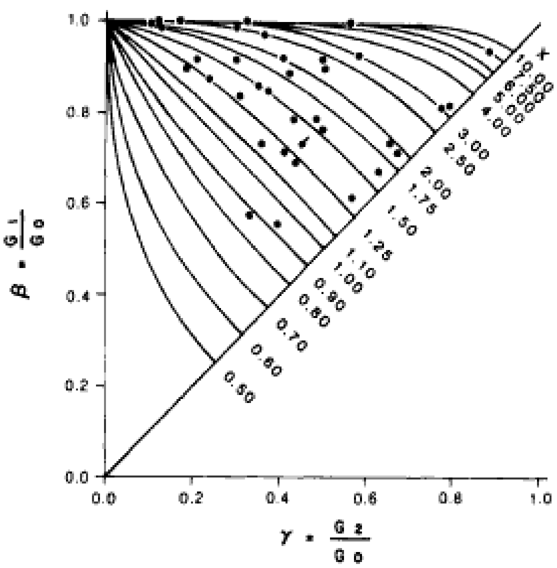


Figure:6- Application of the positive nomgraph to channel width for a miniature drainage network (Eastman. Quebec). The Flag indicates two data points

Bainganga are plotted on the positive nomograph in figure 5. Note that in four cases the width of the receiving stream was smaller than the width of the major tributary but larger than the minor tributary. Thus, only six junctions are plotted on the graph. Adjustments in width are highly variable as shown by the value of x which ranges from 1.1 to a value larger than 7.5. This implies that b derived for a junction is also highly variable. Perhaps because of the variability and small sample size the six data points in figure 4 do not cluster around the expected (2.94) derived from the hydraulic geometry exponent. A larger sample is required to give a reliable estimate of $\overline{x}$ and its reciprocal, the network value of b.

Given a sufficient sample, however, the monograph may be used to estimate the hydraulic geometry exponents in cases where

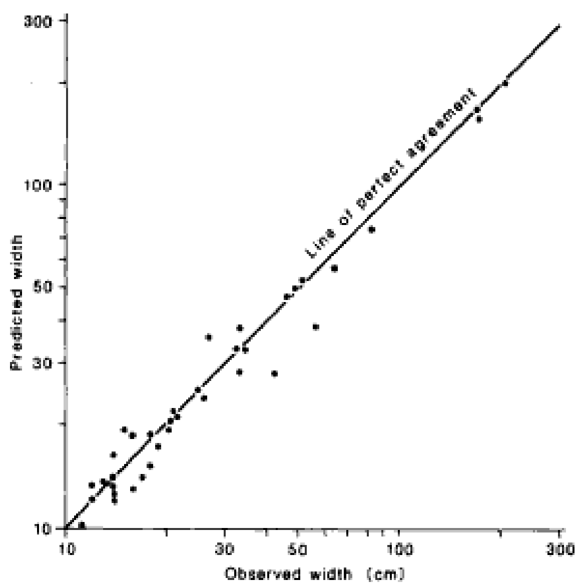
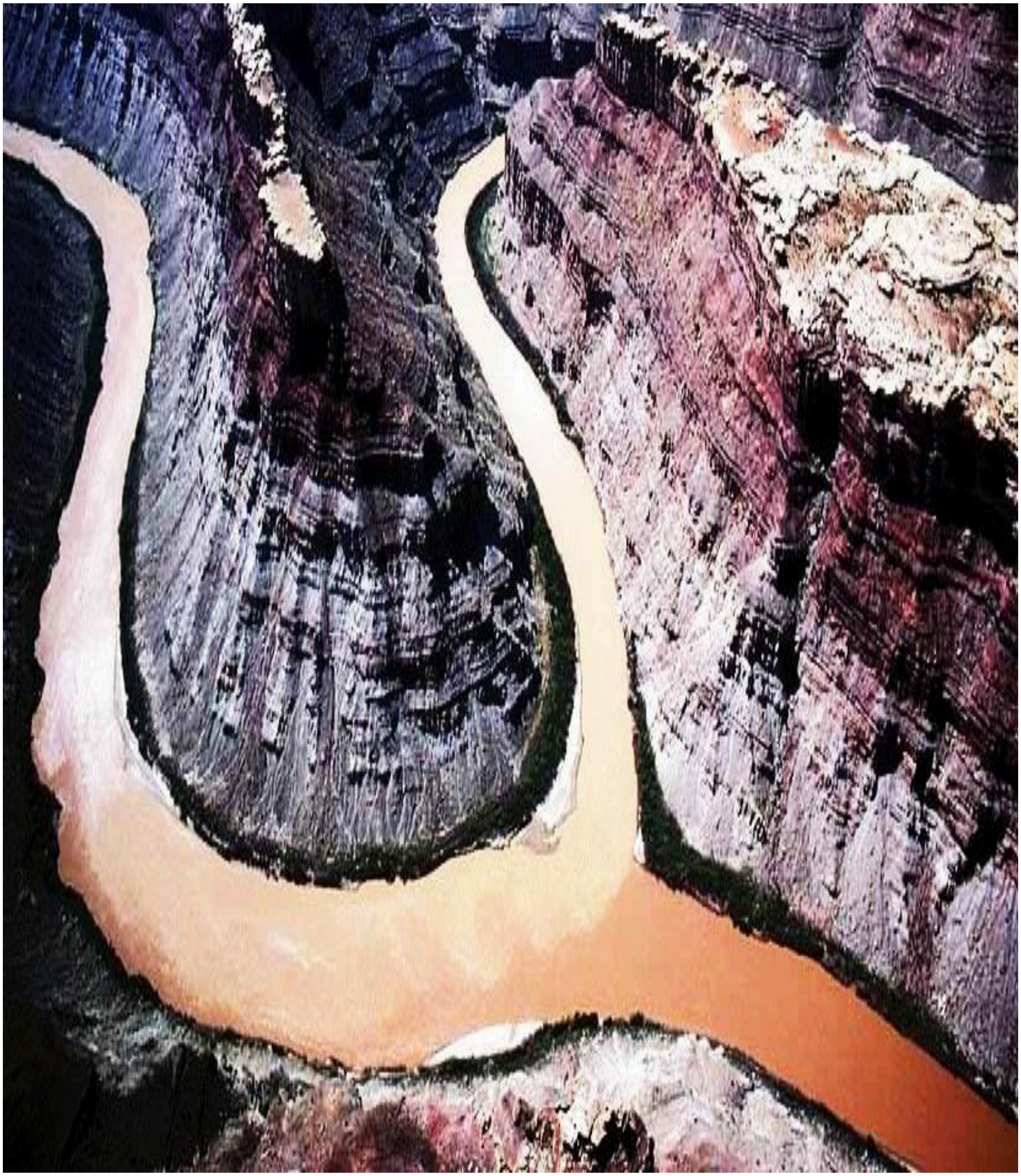


Figure:7. Comparison between observed and expected widths in a miniature drainage network. The model is calibrated using the median junction-specific exponent as the value of b. $\mathbf{b = 56, \overline{x} = 1.8}$



The confluence of River Bainganga and Ghogra in eastern Pachmarhis

Discharges are unknown. To illustrate such an application of the model, I have measured bankfull widths and channel slopes in a miniature ephemeral stream network of approximately 45,000 m^2in area. The site is located in the eastern Pachmarhis (Satpura-India). Thirty-seven junctions with homogeneous bed and bank material were selected and surveyed in the field. Width changes are presented in figure 6. Again, the exponents found at individual junctions are highly variable. The data points are scattered mainly in the upper part of the graph, indicating that changes in width at a junction are minor. The median exponent is 1.8. which suggests that width increases as a function of discharge raised to the .56 power. Using this value of b in equation (7) leads to good predictions of the observed channel widths (fig. 7). Assuming that the continuity equation holds, b could therefore be estimated by application of the nomograph, in spite of the great variability in the values of x.

TABLE 1
Variation of the Median Exponent of Width Changes with Channel Size for Three Junctions in Miniature Drainage

Width Range (CM)	Median Exponent $\bar{x} = (1/b)$	N
$\leqslant 20$	1.60	16
20-40	1.80	9
40-80	1.65	6
$\geqslant 80$	1.85	6

It appears that the variability in morphometric adjustments at a junction is not related to channel size or branching symmetry. In table 1, I show that the median $\bar{x}$ calculated for different width ranges does not change systematically with channel size. I have also looked at the effect of branching symmetry on the values of the exponents. Branching symmetry is usually defined by the symmetry ratio Q_2/Q_1 where $Q_2 \leqslant Q_1$. Because of the transient nature of several small channels and of their divides at the headwaters, it was impossible to evaluate drainage area or link magnitude and to use these variables as surrogate measures for discharge. I have defined the symmetry ratio by W_2/W_1.

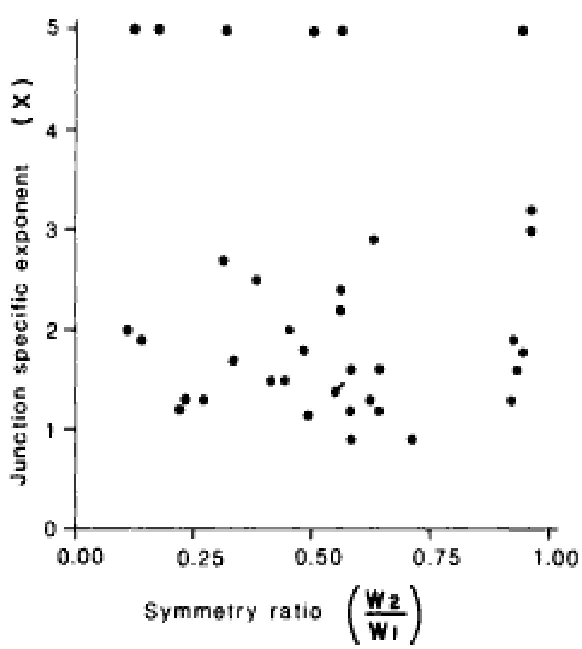


Figure: 8. Relationship between branching symmetry and junction-specific exponents for changes in channel width at a junction as in figure 6.

As depicted in figure 8. The conditional distributions of x given a symmetry ratio, all have similar form and do not exhibit any trend. Variability seems to be inherent to the adjustments taking place at a junction.

Slope data from the miniature network are reported on the negative nomograph in figure 9. The points are scattered widely over the graph. Note that nearly half of the junctions plot on the diagonal of

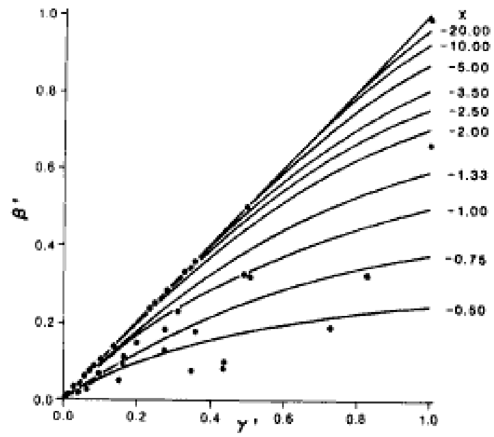


Figure: 9. Application of the negative nomograph to slope data from a miniature drainage network.

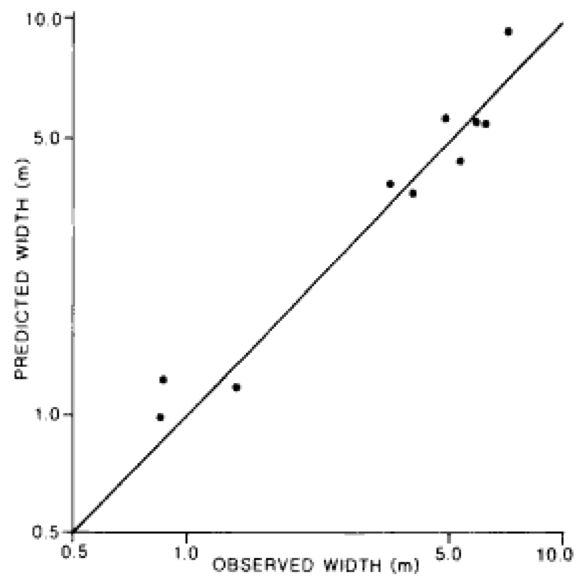


Figure:10. Comparison of expected and observed widths for the River Bainganga using the width-magnitude exponent. $k = .60; \bar{x} = 1.67$

the graph where the exponent is very negative or indeterminate and that four junctions could not be plotted on the nomograph. When a small stream meets a large tributary the mainstream is largely unaffected. Hence, $S_1 \cong S_2$ and the slope ratios $S_1/S_2 = \gamma'$ and $S_0/S_2 = \beta'$are approximately-equal. Thus for an asymmetrical junction equation (12) requires that must be very negative, and therefore z must be less x negative and should approach zero. In cases of extreme asymmetry, the difference between γ' and β'tends to be very small, and x becomes indeterminate. As I see on figure 8, several junctions are in this category. As the flows of the tributaries and their slopes approach equality, the difference between γ' and β' increases, S_1 becomes larger than S_0, and x becomes less negative. Thus symmetry leads to more negative values of z. For the Eastman data, the median value of x is close to - 1.0 and indicates a high rate of change in slope downstream. The long profile of the mainstream is indeed very concave at the headwaters where branching is symmetrical. Further downstream, often the difference in slope between a major tributary and the main stream is negligible and branching is asymmetrical. This explains the two groups that I see in figure 8; one group is along the diagonal, the other below it.

DISCUSSION

The model outlined in this paper is simply deduced from the substitution of the downstream hydraulic geometry relationship into the continuity equation of flow at a river confluence. Richards' model, on the other hand, is based upon the relationship between channel geometry (e.g., width) and drainage network magnitude and it represents an attempt to find an alternative to the classical framework of hydraulic geometry. It is interesting to note, however, that network magnitude also incorporates the notion of continuity of flow at a tributary junction and

$$N_0 = N_1 + N_{2.} \tag{14}$$

Since a good relationship usually exists between width and magnitude, it is therefore possible to substitute equation (2) into (14) and to derive a model analogous to our own

$$W_0^{1/k} = W_1^{1/k} + W_2^{1/k}. \tag{15}$$

Hence, the exponent x in equation (6) may be equal to $1/b$ or to $1/k$ depending upon the point of view chosen by the investigator.

Assuming that water discharge and link magnitude are related by a power function

$$Q = aN^i \tag{16}$$

then the values of b and k are themselves related by

$$k = bi. \tag{17}$$

Thus, in order to have a unique value for x and a single model i has to equal 1.0. At a river confluence, I know that

$$Q_0 = Q_1 + Q_2 \tag{4}$$

$$N_0 = N_1 + N_2 \tag{14}$$

and

$$A_0 = A_1 + A_2 \tag{18}$$

where A is drainage area. All three statements must be true simultaneously. This implies that discharge has to increase linearly with drainage area and link magnitude. Theoretically one expects i to be equal to 1, and x is therefore unique. Data to estimate empirically the value of are scanty, however. Graf (1975) reported that the exponent of the relationship between Q and N is very close to unity (1.05). This value is derived from several basins from Colorado. I do not know if this statement has universal applicability, however.

In the Bainganga River, I found that b was 0.34, and k was equal to 0.67 for the blue line network and 0.60 for the contour-crenulated network. Both relationships between width and magnitude were very strong while the relationship between width and discharge exhibited a wide scatter. Thus, it appears that k is much larger and more reliable than b. First, however, I should see how changes in bank full width at junctions would be affected by inserting into the model (eq. 7) the reciprocal of k for the value of $\bar{x}(\bar{x} = 1.67)$. Predictions of the widths of the receiving stream for the River Bainganga were computed using this new value. As shown in figure 10, the predicted and observed widths are in very close agreement, although the data points are slightly more scattered than in figure 3 where predictions Ire obtained using the reciprocal of the hydraulic geometry relationship exponent ($\bar{x} = 2.94$)

Despite the large discrepancy between k and b both sets of predictions are adequate. Their result is attributed to the lack of sensitivity of the predictions to changes in the value of $\bar{x}$ as the best possible solution is approached. In order to illustrate that problem, I have computed predicted widths for several values of $\bar{x}$ and plotted on a graph the sum of the absolute values of the differences between observed and predicted widths for the 10 junctions of the River Bainganga as a function $\bar{x}$ Although it is not readily apparent from the graph in figure 11, the best overall prediction—which minimizes the sum of absolute differences—is achieved with an $\bar{x}$ equal to 4.05 (b= 0.246). I note immediately that the optimum is very insensitive and that the goodness of fit remains nearly constant as x becomes larger than 3.0. Only when $\bar{x}$ gets smaller than 2.0 do I see a rapid deterioration of the quality of prediction. This rapid increase in the sum of the absolute value of the differences is due to the fact that small changes in $\bar{x}$ in this range of values are translated into large changes in b (and/or$k$) and therefore affects the downstream rate of change in width drastically. On the other hand, as I move from the optimum to the largest values of $\bar{x}$, large changes in $\bar{x}$ cause only minor effects of the value of b (and/or k). This sensitivity analysis reveals that the exponent $\bar{x}$ derived from the hydraulic geometry relationship is closer to the optimum than the exponent computed from k. it is noticeable that $\bar{x} = 2.94$ is

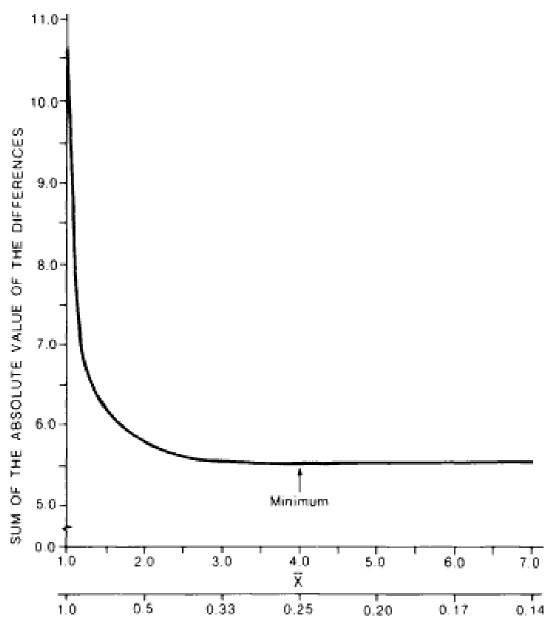


Figure: 11. Evaluation of the goodness of fit between observed and expected widths as a function of $\bar{x}$ for the River Bainganga

Located near the leftmost end of the zone where the goodness of fit remains constant. Using this criterion of fit, b provides a better estimate of $\bar{x}$ than k.

I may be able to explain the discrepancy between the values of b and k. It is apparent from the graphs published by Richards (1980. p. 243) that the slope of the relationship between $W$and N is inflated. Deviations from the regression line are systematic, suggesting that the data should be separated into two equal groups, one for small and the other for large magnitude streams. The value of k for each of these subsets is less than Richards' k calculated for the whole set of data.

The variability in morphometric adjustments taking place at river junctions also raises important questions. According to the model, I expect that each variable will adjust at a confluence in compliance with the hydraulic geometry relationships. The complexity of the processes involved when flows are merging at a junction could be such that adjustments may occur in several variables at once. The hydraulic geometry variables are so interrelated, that if the junction specific exponent of one variable, e.g., width, does not behave as expected from the conventional downstream hydraulic geometry relationship it will cause deviations in the junction specific hydraulic geometry exponents for some or all of the other variables. Thus morphometric adjustments at a junction are often incompatible with the hydraulic geometry statements unless I also look at all the variables involved in the continuity of flows. Junctions may therefore exert an important influence on the downstream hydraulic geometry and partly explain the residuals from the relationships.

Variable adjustments in morphometric at river junctions may also have important implications for the optimal angular geometry models. Roy (1983. 1985) assumed in his optimal branching angle model that average changes in channel form could be adequately described by equation (6). He also suggested (Roy 1982, 1985) that the variability in junction angles could be correlated with the variability in channel adjustments at a junction. In as much as junction angles increase systematically with asymmetry in tributary size (Lubowe 1964; Pieri 1984), while morphometric adjustments at river junctions do not seem to vary systematically with asymmetry, it would appear unlikely that variability in form adjustments at junctions is related to the variability in junction angles. However, the relationship between channel form adjustments and branching angles is probably very complex and deserves further attention.

Finally, the relationship between radius (r) and discharge is described by the power functions relating radius to discharge or to magnitude:

$$r = aQ^{j} = a'N^{k} \tag{19}$$

where it is assumed that $N \propto Q$ and $j = k$: a' is a constant. Therefore

$$Q = \left(\frac{r}{a}\right)^{1/j} = \left(\frac{r}{a}\right)^{1/k} \tag{20}$$

For power minimization in turbulent flow,$j = 3/7$ (Uylings 1977). This implies that velocity increases asQ^{14}, fairly close to the value for rivers. By substituting equation (20) into the continuity equation, I get

$$r_0^{1} = r_1^{1/k} + r_2^{1/k} \tag{21a}$$

and

$$r_0^{x} = r_1^{x} + r_2^{x} \tag{21b}$$

where according to Murray (1926). x Equals 3. Sherman (1981) reviews the biological evidence that $x = 3.0$ as derived from equation (21). I believe that equation (6) represents a general model of morphometric changes at a junction and it may be applied in any tree where the continuity equation of flow holds at a junction.

CONCLUSION

The model proposed in this paper seems adequate to describe average changes in form, especially channel width, at a river junction. The behavior of individual junctions, however, is highly variable and complex. One problem with our model, and with the other models proposed in geomorphology thus far (Miller 1958; Richards 1980), is their extreme simplicity as opposed to the complex processes occurring at a channel junction. Despite the fact that our model is deduced from two I will acknowledged statements, changes in x and thus in geometry at a particular junction remain unpredictable.

The simplicity of this model contrasts strikingly with the complexity of the models developed by engineers. Ibber and Greated (1966) pursued Taylor's (1944) approach based on the change in momentum at a junction. Owing to the complexity of channel processes at a river junction, they only allowed depth to vary and should theoretically that depth should decrease after a junction. Ko-mura (1973) also derived a model of changes in channel depth based on the continuity equation for sediment discharge. These contributions always rest upon very restrictive assumptions which limit their applicability to natural rivers. Thus far, it appears that none of the existing models in the geomorphology and engineering literature is adequate to predict channel changes at individual junctions. One interesting feature of our model is its applicability to cases where the hydraulic geometry relationships are unknown. The average form adjustment represented by the median junction specific exponent may be used to deduce hydraulic geometry exponents for rivers where discharges have not been measured.

References

- GRAF, W. L., 1975, A Cumulative Stream-Ordering System: Geog. Analysis, V. 7, P. 335-340.
- H.J. de Blij, Peter O. Muller, and Richard S. Williams, Jr. (2004) Physical Geography: The Global Environment Oxford University Press Sales Representative.
- KOMURA, S.. 1973. River-Bed Variations At Confluences: Proc. Int. Assoc. Hydraulic Res. Paper A66. P. 773-784.
- LEOPOLD. L. B., And MADDOCK, T., JR., 1953, The Hydraulic Geometry Of Stream Channels And Some Physiographic Implications: U.S. Geol. Survey Prof. Paper 252, 57 P.
- LUBOWE,. J. K., 1964, Steam Junction Angles In The Dendritic Drainage Pattern: Am. Jour. Sci., V. 262. P. 325-339.
- MILLER, J. P., 1958, High Mountain Streams: Effects Of Geology On Channel Characteristics And Bed Material: New Mexico Bur. Mines Min. Res. Mem. 4, 52 P.
- MOSLEY. M. P.. 1976. An Experimental Study Of Channel Confluences: Jour. Geology, V. 84, P. 535-562.
- MURRAY, C. D.. 1926. The Physiological Principle Ol" Minimum Work. I: Proc. Nat. Acad. Sci., V. 12. P. 204-214.
- PARK. C. C 1975. Stream Channel Morphology In Mid-Devon: Trans. Devonshire Assoc. Adv. Sci.. V. 107, P. 25-41
- Poppe de Boer, George Postma, Kees van der Zwan, Peter Burgess, Peter Kukla (2009)Analogue and Numerical Modelling of Sedimentary Systems: From Understanding to Prediction International Association of Sedimentologists.
- PIERI. D. C. 1984, Junction Angles In Drainage Networks: Jour. Geophys. Res., V. 89. P. 6878-6884.
- RICHARDS. K. S., 1976, The Morphology Of Riffle-Pool Sequences: Earth Surface Proc.. V. 1. P. 71-88.
- Roman S. Chalov (2008) Channel Processes in the Rivers of Mountains, Foothills and Plains Institute of geography and spatial management of the Jagiellonian University,
- R. E. Criss, David Alexander Wilson (2003) At the confluence: rivers, floods, and water quality in the St. Louis region Missouri Botanical Garden Press,
- RICHARDS. K. S., 1977. Channel And Flow Geometry: A Geomorphological Perspective: Prog. Physical Geog.. V. 1, P. 65-102.
- RICHARDS. K. S., 1980. A Note On Change In Geometry At Tributary Junctions: Water Resources Res., V. 16. P. 241-244.
- ROY. A. G.. 1982. Optimality And Its Relationship To The Hydraulic And Angular Geometry Of Rivers And Lungs: Unpub. Ph.D. Dissertation, Stale University Of New York At Buffalo.
- ROY. A. G.. 1983, Optimal Angular Geometry Models Of River Branching: Geog. Analysis, V. 15, P. 87-%.
- ROY. A. G.. 1985, Optimal Models Of River Branching An- Gles, *In* WOLDENBERG, M. J., Ed.. Models In Gcomorphotogy. *In* Binghampton Symposia In Geomorphology 14: London, George Allen Un- Win. P. 269-285.
- TAYLOR. E. H.. 1944. Flow Characteristics At Rectangular Open Channel Junctions: Trans. Am. Soc. Civil Eng., V. 109, P. 893-903
- WEBBER N. B. And GREATED, C. A., 1966. An Investigation Of Flow Behaviour At The Junction Of Rectangular Channels: Inst. Civil Eng. Proc. Paper 6901. P. 321-334.
- WOLDENBERG. M. J.. 1972. Relations Between Hor-Ton's Laws And Hydraulic Geometry As Applied To Tidal Networks, *In* Harvard Papers In Theoretical Geography 46: Springfield. VA. Clearinghouse Fed. Sci. And Tech. Infmtn, U.S. Dept. Commerce (Order No. AD 744043).