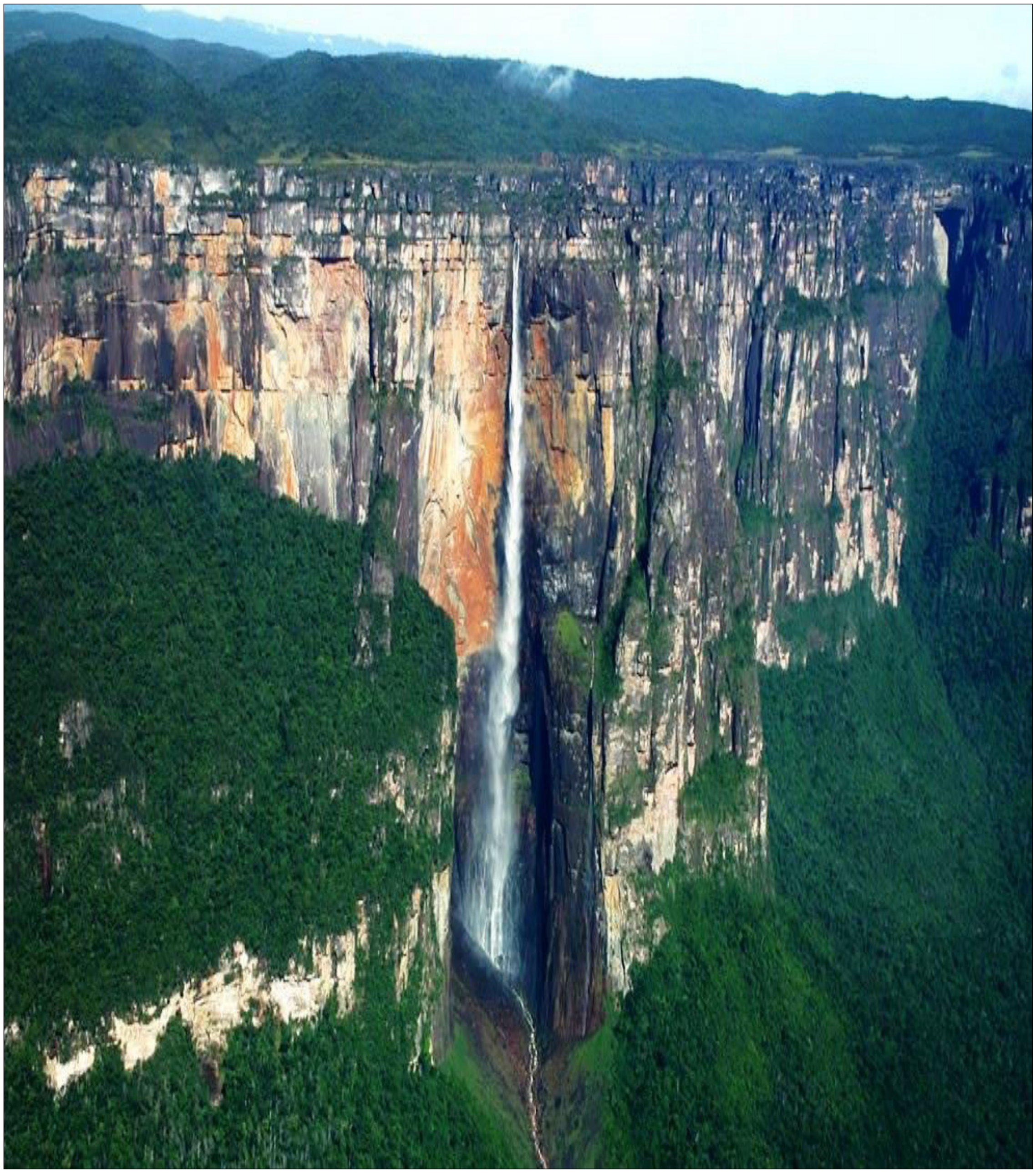
Temporal changes in the Pachmarhi

Dr. N.L. Dongre



In order to explain the Morphological development of the Pachmarhis, it is useful to establish a model against which the present landscape surface can be compared. Since this up warp occurs in an area, whose planation surfaces have been warped. It means that any surface formed prior to the initiation of the up warp will be distorted as the tectonic deformation took place, and the rivers incised into the surface as soon as further uplift continues. Planation however continues so long as there is a period of tectonic quiescence of sufficient duration for the surface to develop in between two successive uplifts and as long as the rate of drainage incision keeps pace with the rate of uplift along the tectonic axis.

If however the latter becomes excessive the river tends to reverse. Since the Dhupgarh surface had a radial slope in relation to the axis, allowance must be made for this; if however it is assumed that the slope of this surface was as much as $35\ meter/\ kilometer$ the total uplift would be $300 meters$. The lowland by contrast shows an opposite rise. Thus it would be necessary to invoke a shift of the axis of up warping by some $24\ kilometers$ to the south, to lie now near the Deccan Trap cliffs.

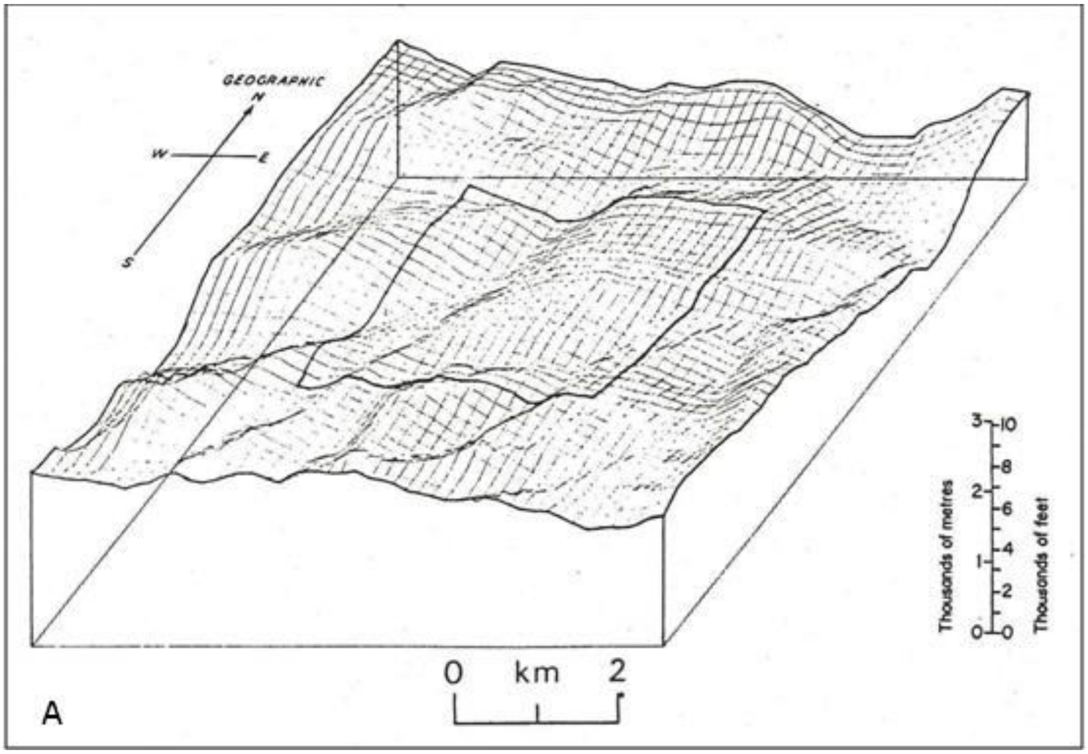
Drainage can be expected to develop in an area such as this, where relief barriers can be fixed by shallow dips, and where uplift with consequent rejuvenation tends to draw the rivers across the east-west structural grain, but the process remains questionable. The water gaps, and wind gaps too are sufficiently aligned and independent of the transverse structure to disqualify the explanation of regressive erosion and capture, and crest beveling was not as advanced as to allow major rivers to swing across the interfluves. It however, seems possible that the drainage evolved from a combination of part inheritance and limited superimposition at the close of the Pachmarhi leveling, when the southward shift of the axis of up arch assisted in drawing the drainage lines across the low relief barriers of the Dhupgarh Surface. It is plausible that the streams were inherited from an earlier planation surface, the Dhupgarh surface and the final leveling as a result of the pedimentation of the Pachmarhi Surface, when relief barriers were subdued enough for the rivers now rejuvenated to be superimposed on the lower and surfaces below. Based on these considerations a scheme of events leading to the morpho-evolution of the Pachmarhi has been worked out. It is summarized in Table 1. Stage wise development of the morphology is schematically depicted in Figure 1 A, B, C, D, E.

Relict planation surfaces are found to be the most potent guides in formulating the sequence of event. In addition they strongly indicate the activeness of the area that had been hitherto regarded as a part of an eventless stable landmass which is further corroborated by the(A) presence of numerous waterfalls both cascade and other types, wind and water gaps at high elevation, deep gorges carved out by perennial streams, numerous springs, Karst-like topography including subterranean drainage, caves and caverns etc;

Pachmarhi morpho-evolution chronology denudational chronology

Stage	Characteristic Feature	Probable age
6	Development of areas locally lower than the lowland surfaces; slow intermittent uplift of the area.	Pleistocene.
5	Development of lowland surface into extensive plains with outliers of the upland surfaces; discordant drainage lines, extensive rivers capture; surface develops along a feather edge by scarp retreat as basal.	Pliocene to Pleistocene.
4	Development of 1,100 m upland surface as intermont plane by retreat of the residual scarps of 1,300 meter plane; landscape with early mature features; river development advanced; centripetal slope with fringing uplands result in fairly co-ordinated drainage with ancestral drainage lines; persist slow uplift in recurrent phases resulting in several ledges on the scarp.	Oligocene.
3	Resurrection of the pre-trap surface, initiation of 425 meter	Lower Miocene (?)

	planation phases; rivers develop on either side of the major axis of up-arching ; initially unstable	to Lower Eocene.
2	Land surface covered completely by the Decan Trap plateau landscape drainage on the Cretaceous plane completely covered intrusion of dolorite dykes	Lower Cretaceous to Lower Tertiary
1	Formation of the Pre-trap Cretaceous peneplane on the Gondwana sedimentaries.	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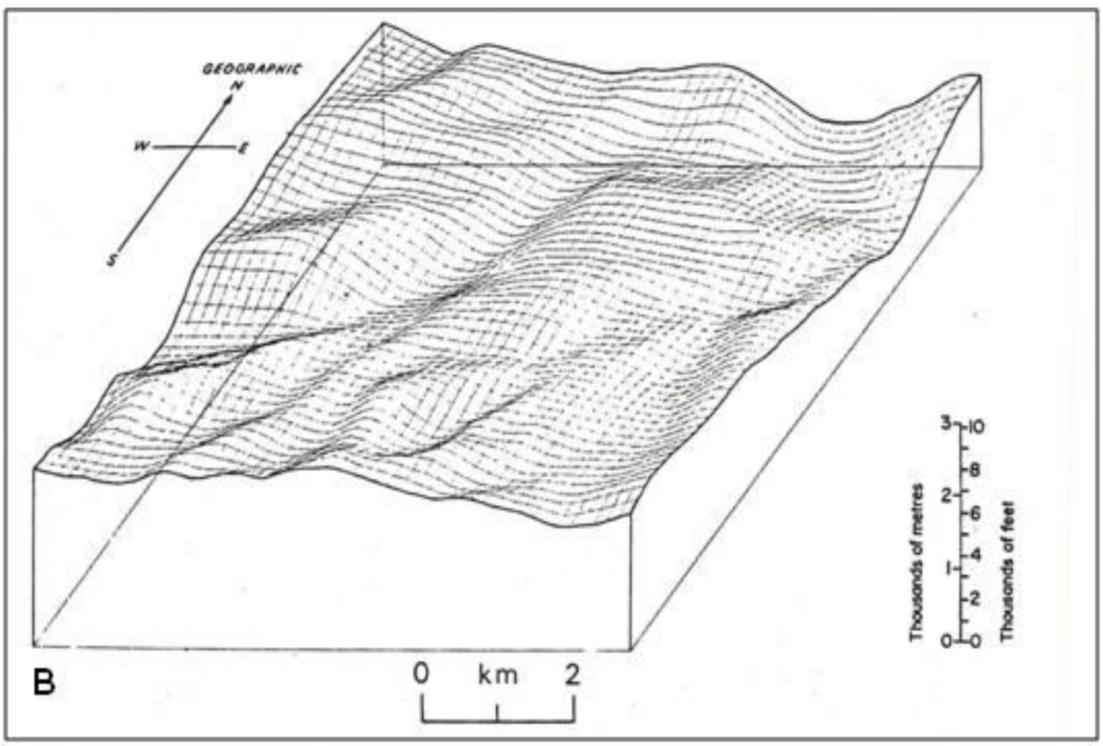
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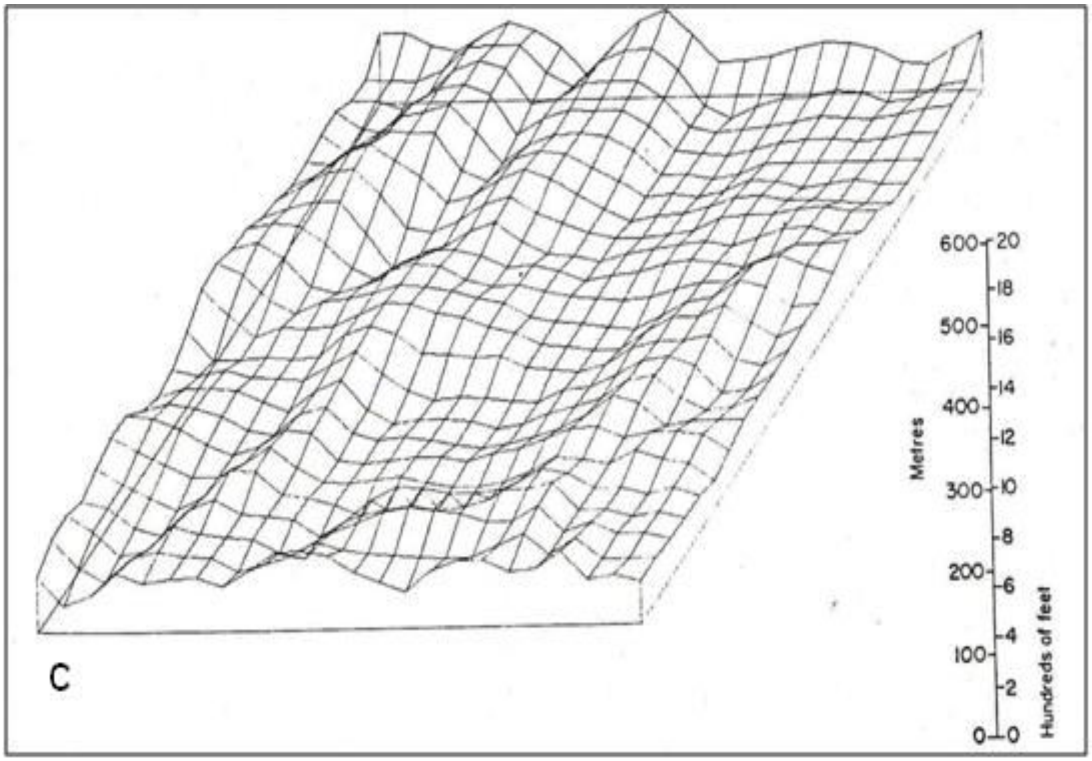
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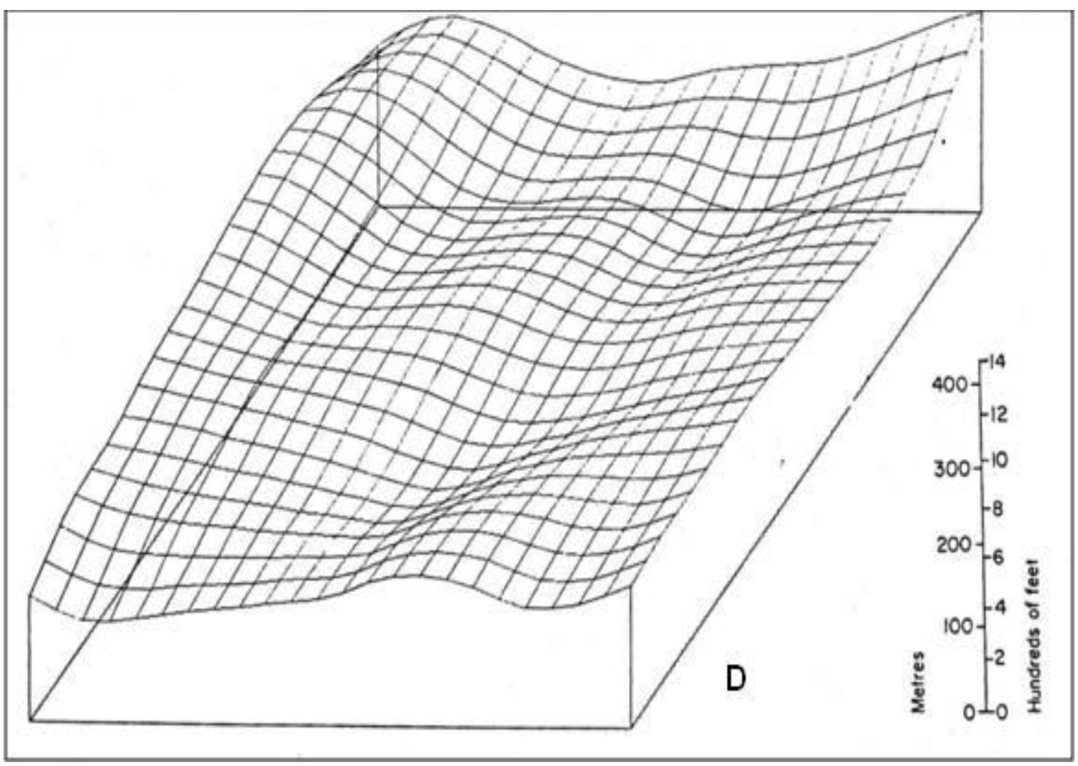
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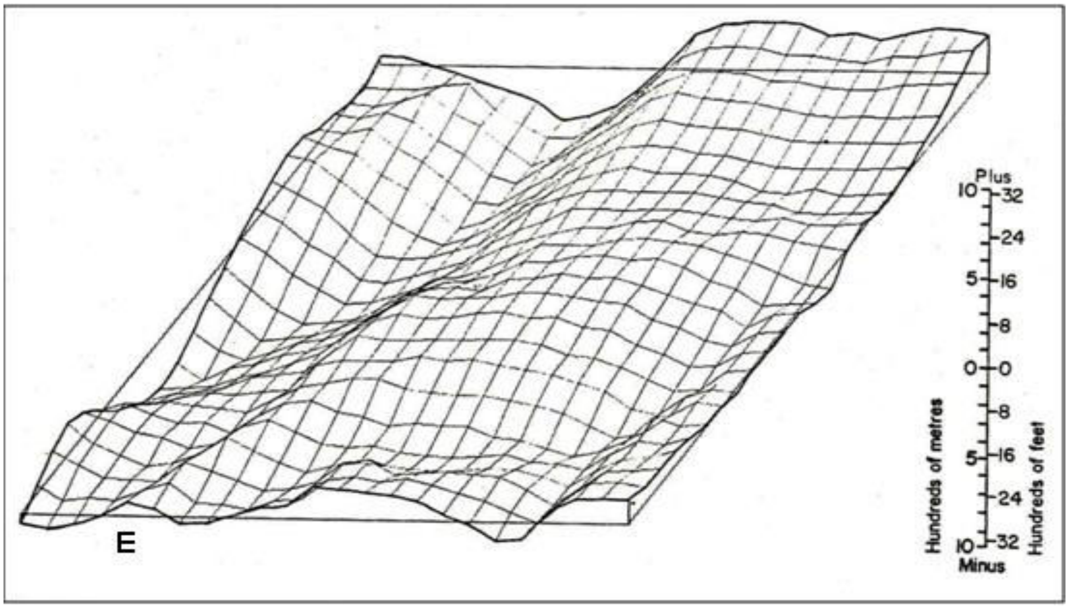


Figure 1. A,B,C,D,E Block diagram of Pachmarhi altitude matrix by program ISE1174/15P4/MATCON , to illustrate the major stages in the evolution of the Pachmarhi(A) Satpura Basin Gondwana Resurrection: The Gondwana exhumation ... Post-Trap uparching resuits in the denudation and progressive uncapping of the Deccan Traps. Streams flow radially around the E-W trending axis,(B) Dhupgarh Planatlon: The upland Landscape Cycle begins ... equivalent to the Dhupgarh planation. This surface was diversified by relative amplitudes of relief due to incomplete planation. Drainage lines persist with very little adjustment to structure,(C) Pachmarhi Planation: Regional uplift results in incision of streams into the Dhupgarh Surface; a new cycle is initiated, which evolved into the Pachmarhi Surface. The land surface reaches a stage of late maturity, with fringing Dhupgarth remnants, Drainage is still radially disposed around the Upland residuals,(D) Bijori Planation Begins : More uplift and differential warping (?) ... the tectonic axis shifts south by some 24 kilometer, a new cycle of planation is initiated alone the feather edge of the north dipping Upper Gondwanas on which is developed the Upland Landscape. The resurrection of the Lowland cycle is started by the retreat of the Pachmarhi scarp. Drainage further isolates the Upland remnants and by superimposition, discordance sets in as in the major rivers such as the Denwa and Sonbhadra,(E) Bijori Planation: Scarp retreat advances, developing the Bijori Surface at the expense of the Upland Landscape.

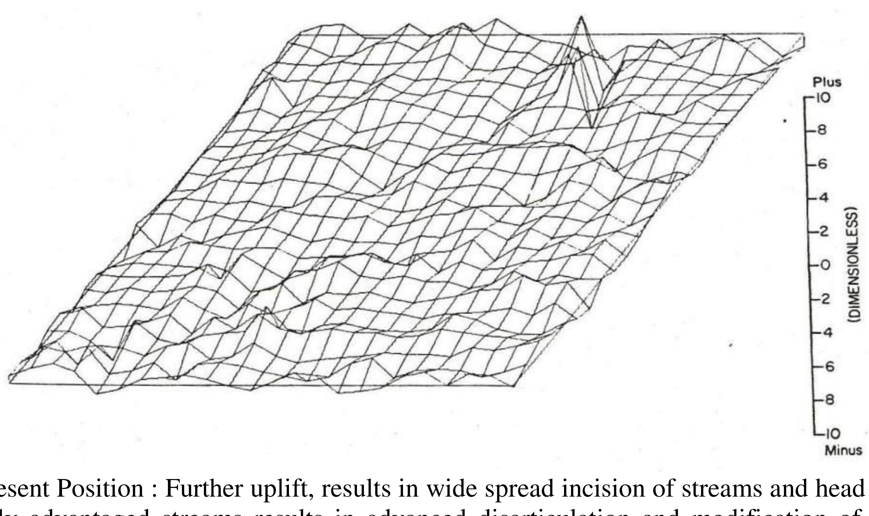


Figure 2 . Present Position : Further uplift, results in wide spread incision of streams and head ward erosion of tectonically advantaged streams results in advanced disarticulation and modification of the extended drainage net- work.(Block diagram of Pachmarhi altitude matrix by program $ISE1174/15P4/MATCON$)

(b) Complete absence of flood plains.

Thus above features bear testimony to the youthful character of the area. Further expansion of the Lowland Surface is particularly noticeable. In this connection possibly a southward shift in the axis of the domal up-arching resulted in the warping of the Upland surfaces and caused the limited superimposition and discordance of the major drainage lines, which lead to the formation of the Lowland surface (exhumed by the uncapping and denudation of the Deccan Trap Lavas) evolving at the cost of Uplands, being consumed due to scarp retreat.

Parallel scarp retreat

A survey of published literature pertaining to scarp retreat, notably the studies of Budel (1965) Budel (1973), Cotton (1973), King (1976), and Ollier (1960) shows that in humid climates scarp-retreat is pronounced where stream flow is forceful and the denuded material is transported quickly away from the pediment (foot-slope) by a basal stream. In the pachmarhi area, stream flow is forceful, in fact the entire section of the Denwa river from Binora to Sukhadongar is pot-holed (Figure 3) and characterized by numerous rapids. (Figure 4)The present phase of stream erosion seems to be one of incision. This results in over-steepening of the scarp-foot slopes and hence should accentuate scarp-retreat, as evident from the pedologic observations given below.

So far the discussion has been confined to the processes of recession of scarps. It is now important to digress from processes that cause scarp retreat to examine evidence that supports parallel retreat of the Pachmarhis Scarp. Several lines of evidence from field and map observation indicate parallel scarp recession. Of these, the following two are deemed most important.

A. Pachmarhi sandstone outliers: These sandstone outlier (Buttes) are often aligned along trends that are parallel to the present margins of the Pachmarhi Plateau. Examples of these outliers are the Mankideo, Barghat, Kedardeo (Figure 5) Burimai etc. The farther they are from the scarp smaller is their areal extent and elevation. This indicates increasing relative age of isolation from the plateau and the once continuous extent of the Upland Landscape. Using these outliers, to reconstruct the Pachmarhi Surface by extrapolating form lines and contours across the intervening embayment.

B. Rectangular drainage pattern : Namely, the similarity of drainage patterns far away from the present scarp to those present very near the scarp and to those present on top of the Pachmarhi plateau. The rectangular stream segments are laid-off herringbone fashion to the north-south drainage divide developed on the Lowland Landscape. The Denwa now at the base of the Pachmarhi scarp is parallel to the Katha and Bija rivers successively south from the Denwa. In similar fashion, the Bori Nadi now at the base of the scarp is parallel to the Sagum, Kabra, and Sonbhadra rivers successively south. The $E\ N\ E$ aligned interfuves to the above streams are often straddled by the Pachmarhis sandstone outliers described above. (Figure 6).East-west segments are longer and better developed and appear to be parallel with dolerite dykes (Auden 1949) and a major joint trend that characterizes the entire Satpura Basin. It is possible that the $E\ N\ E$ trending pre-existing "Weak zone" is readily exploited by streams draining the plateau, and in due course of time are superimposed onto the Bijori Shales (Lowland Landscape) as the scarp retreats. This results in the rectangular drainage pattern with $E\ N\ E$ elongate segments being parallel to the scarp.

Figure 3. Numerous Potholes are in the Pachmarhis formed by Denwa River.

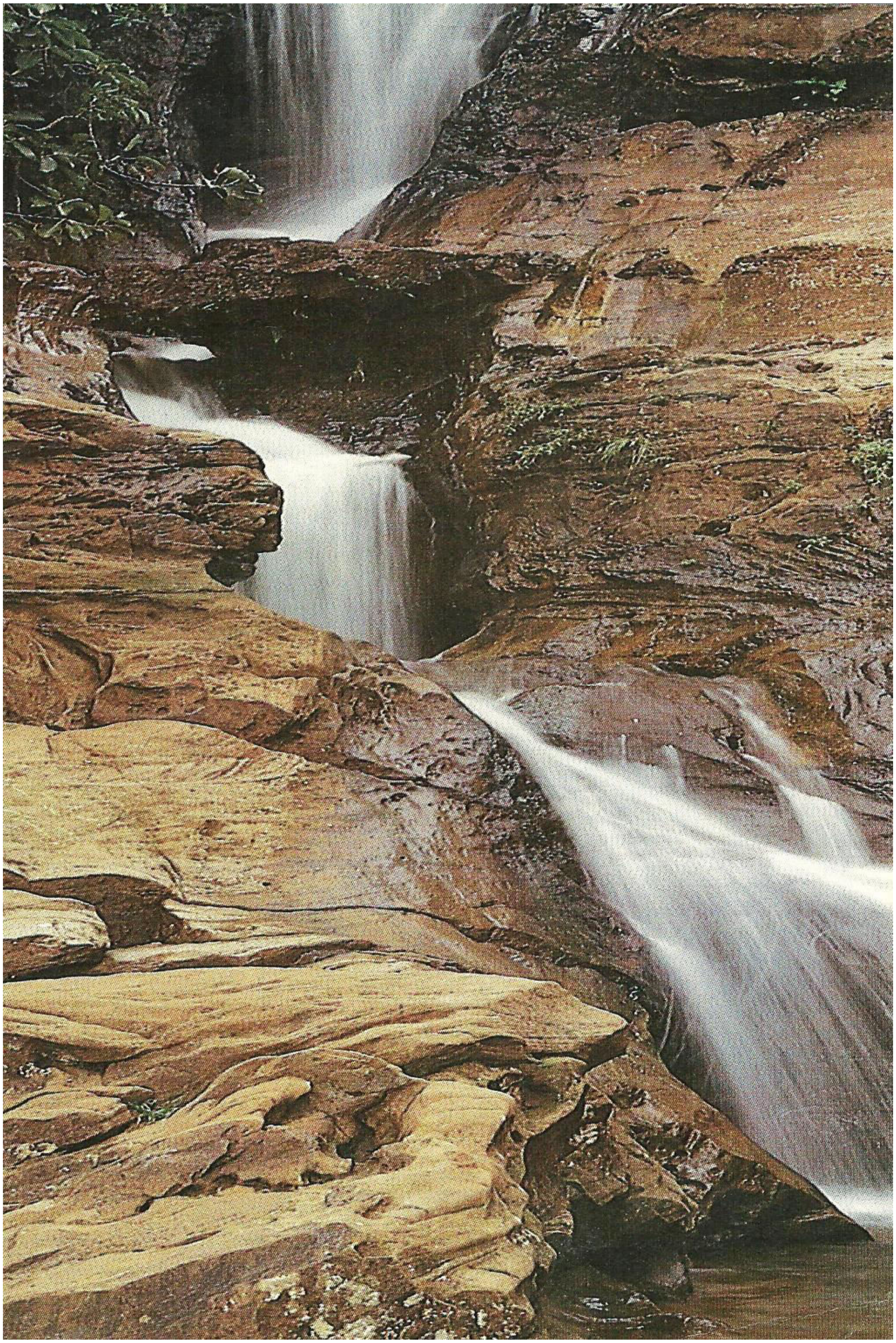


Figure 4. Denwa river, characterized by numerous rapids during its drainage course.

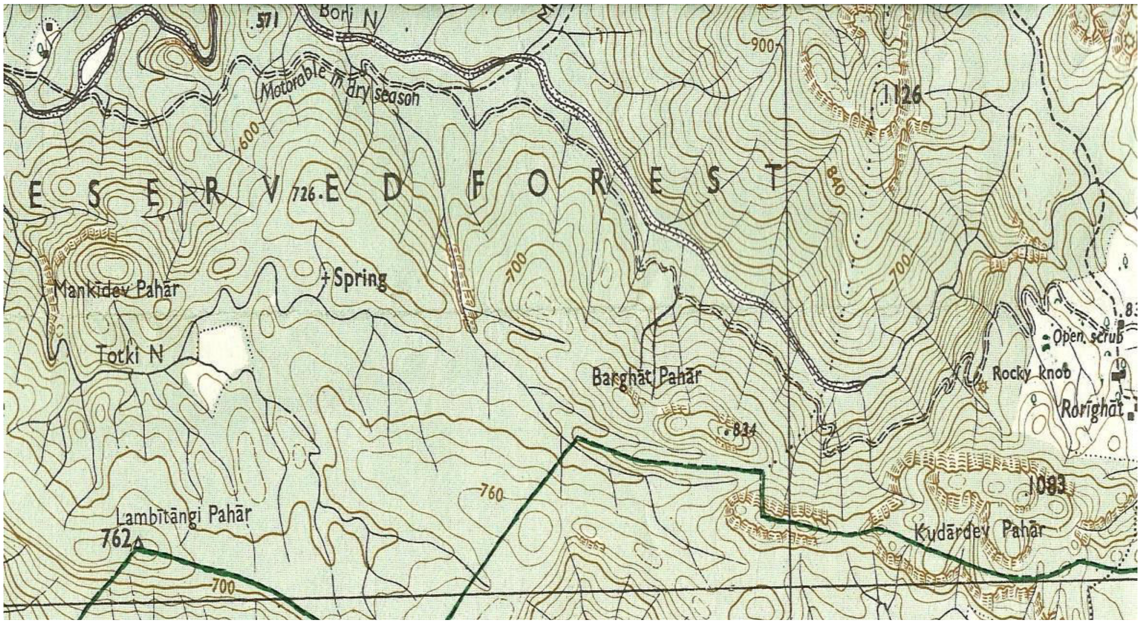


Figure 5. The Mankideo, Barghat, Kedardeo outlier (Buttes) is often aligned along trends that are parallel to the present margins of the Pachmarhi Plateau.

The question now is to explain the presence of the major drainage lines that are discordant with not only the scarp, but the entire structural grain of the Satpura Basin. In fact, the Denwa, Sonbhadra, Dudhi, and Nagdwari etc., originate in the Lowland Landscape south of the Pachmarhi Scarp, and flow directly into the Pachmarhi Plateau, across the axis of the Satpura Dome and emerge north to debouch into the Narmada rift valley. These major rivers could hardly be superimposed from higher planation levels, for there is nothing to suggest that the Deccan Traps Plateau to the south of the study area was ever higher than the Satpura Dome. Moreover, it does not seem possible for a minor stream on the northern flanks of the dome to be drawn across the domal axis by head ward erosion and capture streams on the Lowland Landscape. The only plausible hypothesis, and one that fits the field evidence is that the major streams antedate the domal uplift and were able to maintain their courses across the uplift to the local base level in the Narmada river rift valley.

From the moment the streams were initiated, they have laterally planated and vertically eroded the valley they occupied as a function of the rates of uplift, changes in local base level or presence of contrasting lithologies of differing resistances to erosion (Verma,1972) . The recurrent and often spasmodic nature of uplift and stillstand in the Pachmarhi area has been preserved in the "stepped landforms". (Venkatakrishnan, 1975) Venkatakrishnan 1977). The recurrent nature of tectonism in Central India, especially along the Narmada-Son lineament (Choubey, 1971) has recently been documented in geophysical studies as well (Qureshi and Warsi,1980)

Some field examples

The Narmada rift valley has acted as a rapidly falling and rising local base level to streams that drain into it Dixey (1968) and Vaidyanadhan (1977). The post- Deccan Trap regional slope was to the north into the Narmada rift valley. Drainage lines established themselves flowing northwards possibly due to a greater rate of fall in the local baselevel as compared to smaller rates of uplift of the Satpura Dome. Streams with initially steeper profiles

(Denwa, Sonbhadra etc.) were able to maintain their course across the recurrent uplift where as almost all other streams particularly minor tributaries were defeated. The incompetent streams occupied the structural weak zones that were discussed earlier.

Present day field evidences indicate that several of the dip-streams on the northern slopes of the Satpura Dome, have however, been drawn across the uplift axis to breach initially high divides, break free of the scarp, and establish themselves south of the scarp as basal streams on the low-land Landscape to undermine the scarp itself. An excellent example of such head ward erosion and stream capture is exhibited by the headwaters of the Nagdwari River near the village of Kajri. The east fort of the Nagdwari (the Jambudeep), on the other hand, has occupied a $E\ N\ E$- weak zone and in the process captured several headwater streams draining the north slopes towards Matkuli. Several similar examples can also be seen in Nimdhana, Barkachhar, Somgarh, Deogarh,Dader,Bauta, Jhandiwali Paharia, Guriadeo, Phasi, Guttideo, Supdonger, Patalkot, Pratapgarh, Chandimai, Tamia , Kalapahar , Patarkot, HathiPahar, Dhupgarh, Belkandhar, Jambudeep and Jatashankar. Mahadeva Peak and Chauragarh Peak are postulated to evolve into large outliers near the Pachmarhi Plateau. In similar fashion the $N\ E$ -trending anti-dip segment of the Denwa eroding head wards should breach the scarp in an area cast of Rorighat isolating Kedardeo Pahar and Mankideo Pahar as large outliers separated from the Pachmarhi Plateau by the intervening Bori and Denwa drainage lines.

It is interesting to note that in all the above cases, stream erosion continuously back wears the scarp; periodically sandstone buttes are isolated as outliers once the backwearing stream presumably grades itself to the Lowland level. In all these cases the outliers and streams are aligned parallel to the associated Pachmarhi Scarp.

Estimate of time of initiation of pachmarhi scarp

Scarp retreat takes time, and in the study area where recurrent uplift and consequent stream rejuvenation has resulted in periodic stream incision scarp retreat was by no means uniform. Rather it can be postulated to have occurred in discontinuous phases of rapid recession during periods of still-stand (lateral planation) and slower recession during periods of uplift (vertical erosion). To avoid these spatially and temporally varying circumstances, it is necessary to calculate recession rate within the constraints of several simplifying assumptions namely that, (1) Uplift is continuous and occurs at a constant rate, (2) Scarp recession occurs at a constant rate, (3) To achieve (2) above, lithology of Pachmarhi Sandstone should be considered uniform, and (4) Climatic regime has more or less stayed uniform and humid throughout.

Admittedly, these assumptions are geologically uniformitarian. However, for long-term calculations of scarp recession one may assume uniformity in the geologic variables, with scarp retreat occurring continuously. The present day, Pachmarhi Scarp is at an average distance of $24\ kilometers$ from the outer margins of the Satpura Basin. This figure is obtained by projecting the Pachmarhi Sandstone-Bijori Shale contact and Deccan Basalt-Dhupgarh Surface contact south to intersect along the feather-edge of the Pachmarhi Sandstone outcrop. A detailed search of pertinent literature yielded a range of recession (parallel retreat) rates varying between (a) $1\ meter/1000\ years$, and (b) $2\ meters/1000\ years$ Young. (1974). These figures have been chosen to represent minimum and maximum values of recession of the Pachmarhi Scarp. Extrapolation of this recession range over the estimated $24\ kilometers$ of scarp recession should provide a rough figure for the time of initiation of scarp retreat and hence a maximum age (of exhumation) for the Bijori Surface. Thus, at a recession rate of (a) 1m/1000 years and, (b) 2m1000 years, time of initiation works out at (a) $12\ million\ years$ (U Miocene) and (b) $24\ million\ years$ (U. Oligocene) as the time of initiation of scarp retreat.

Figure 6 . Drainage patterns far away from the present scarp to those present very near the scarp and to those present on top of the Pachmarhi plateau. The rectangular stream segments are laid-off herringbone fashion to the north-south drainage divide developed on the Lowland Landscape.

This figure is no more than an estimate but it yields a valuable number to set landform evolution in the Pachmarhi area within an effective time frame. For this gap to be filled, there is need for a great deal more of geomorphological mapping and descriptive geomorphology.

The preceding study clearly illustrates the intimate relationships between recurrent tectonism, drainage pattern development, and scarp retreat as a result of a complex and hitherto little understood interplay of movements in the Narmada rift valley and uplift in the Satpura Hills of Central India. Chronological data for scarp retreat and drainage evolution are as yet equivocal. But the very periodicity between the continuous process of stream erosion and alignment with occasional isolation of sandstone outliers is worth further research.

A very important time constraint for the beginning of landform evolution within the Satpura Basin is provided by the exhumation of Gondwana sediments due to the uncapping of Deccan Trap basalts. This exhumation marking the beginning of the post-Deccan Trap erosion cycle probably commenced immediately after the erosstion of Deccan volcanism (+ 40 m.y.?). In any event, the pre-Deccan Trap erosion surface called Cretaceous peneplain by Dixey (1968),pre-Trap Surface by Choubey (1971) and Dhupgarh Surface is probably equivalent to the inter-continental Gondwana or African Surface of King (1953) and his later classification as the Mooreland Surface (King, 1976) . It is possible that Dhupgarh planation must have proceeded rather rapidly because of the deeply weathered mantle that characterized the Pre-Trap erosion cycle. Examples of this material can be observed atop the Chauragarh Peak, Burimai Peak, Kedardeo Pahar and along the Deccan Cliffs at Tamia. Once the deeply weathered mantle is removed, formation of subsequent planation surfaces and the Pachmarhi Scarp is much more difficult, for erosion has to proceed on fresh, hard rock.

These time constraints may be used as a means of order of magnitude estimate of denudation chronology in the Pachmarhi area, since no better data is presently available. Crook Shank (1936) convincingly illustrated the intimate relationship between crustal instability and land from evolution in peninsular India. However, the degree to which rivers control and are controlled by tectonics, remains to be worked out and the elucidation of the interplay of geomorphology and tectonics will require much geologic research and mapping.

Geomorphic analysis of irs -1c pan stereoimage:

Now-a-day's the geomorphological mapping is being carried out all over the world efficiently by Remote Sensing techniques. The study area is a part of the northern Satpura of Madhya Pradesh. The images are covered in path 98 and row 56 of $IRS - 1C$ reference map. In SOI Map, the area falls in between latitude $22°20'45''N$ to $22°38'45''N$ and longitude $77°41'45''E$ to $78°51'15''E$ (Figure 7, Figure 8).

Stereoscopic analysis of the images reveal that the morphology is controlled by structure and the typical landforms developed are dissected plateau, pediment, linear ridges, joints, mesas, buttes and deep erosional escarpments, which are common on basalts. The exaggerated stereoscopic view allows identification of various layers of basalt, because of depth perception. The landforms noticed here are dissected plateau. The sub-division is based on the intensity of the dissection of the terrain and altitude.

Quantitative deterministic model study

We are introducing the notion that events occurring in time could be regarded as having varying degrees of memory'. Models for the Pachmarhi, in which events in time are completely independent of all previous events were said to exhibit complete randomness; very few situations in nature really have this character. A continuous random series, while being useful as a concept in an abstract sense, is virtually unobtainable. At the other extreme, all events are entirely prescribed and the system under observation is assumed to have an infinitely long memory. These are the deterministic models of the type to be examined the Pachmarhi landmass in this article and the methods used for their examination, development and testing are those of applied mathematics, especially the differential calculus. Differential equations, with all their ramifications and generalizations, are undoubtedly the most powerful tool in applied mathematics and it is hardly surprising therefore that the models used take on this form.

It is useful at this point to recall the reasons for adopting deterministic mathematical models relating process to response, input to output, and cause to effect. The first is that provided our conception of the process under study can be transformed into the language of mathematics, then there exists a whole system of techniques and a body of theory for manipulating the relationships in the model in an objective and replicable manner. In other words, the procedure offers great facility, provided that, we are familiar with the language and technology of applied mathematics. This procedure of transforming a problem, performing some kind of operation on it and then reversing the transformation is the very essence of mathematical technique, as well as the reason for its adoption in geomorphological research.

The great progress of classical physics in the first part of the nineteenth century, stemming from Newton's law of gravitation and fully developed by laplace and lagrange, depended entirely on simplification and abstraction and rested in the belief that the universe was rationally constructed. The fundamental proposition, that abstract models constrained only slightly by the limitations of experience were of the essence, still holds today even in those sciences such as geomorphology where mathematics has been relatively recently applied.

A second major reason for adopting an applied mathematical approach to process-response modeling is the adoption of a systems approach in the subject at large. While there is still a widespread and elementary view, to some extent propagated in recent literature, that the 'Systems Approach' consists rather largely of 'organizing' things in boxes or of making broad and not very useful statements about interactions, quite the converse is true. This attitude seems to stem largely from the difficulties of accepting the engineering and applied mathematical techniques in areas, not traditionally mathematically-based, such as sociology and human geography. Unhappily, geomorphology still lies in part, at least, in this camp, though books such as Chorley and Kennedy's Physical Geography. A Systems approach have gone some way to improving the situation.

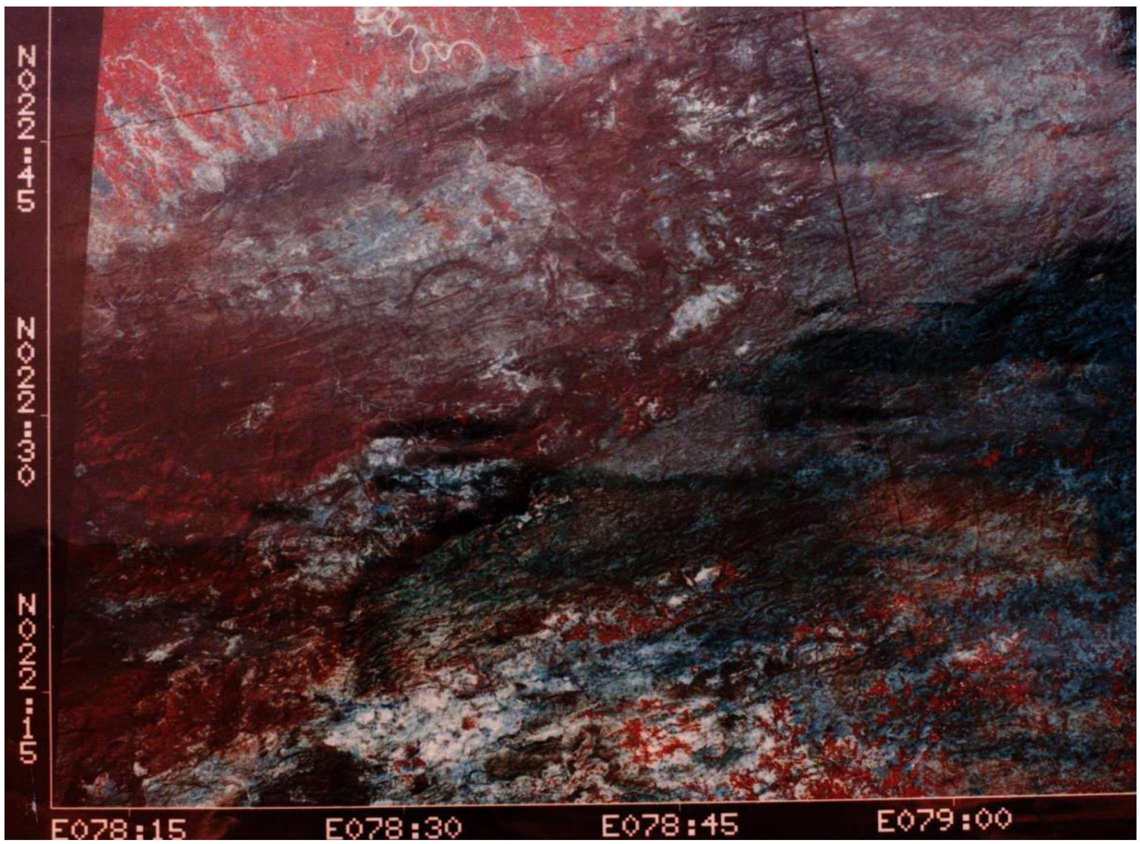


Figure 7. PAN Stereo Images depicting the Pachmarhis

Given a simple input or cause or process in a system, this may, be transformed by a transfer function into an out effect or response "Mathematically, the most simple system could take the form $Y_t = C.X_t$ where X_t and Y_t are the input and output respectively at time t, C the transfer function. In a most general fashion, three typical problems arise: (1) given the input and transfer function determine the output, (2) given the output and transfer function find the input and (3) given the input and output find the transfer function. The systems are usually described in terms of differential equations which almost invariably involve a time element and of course, the transfer function embodies the characteristics of the system.

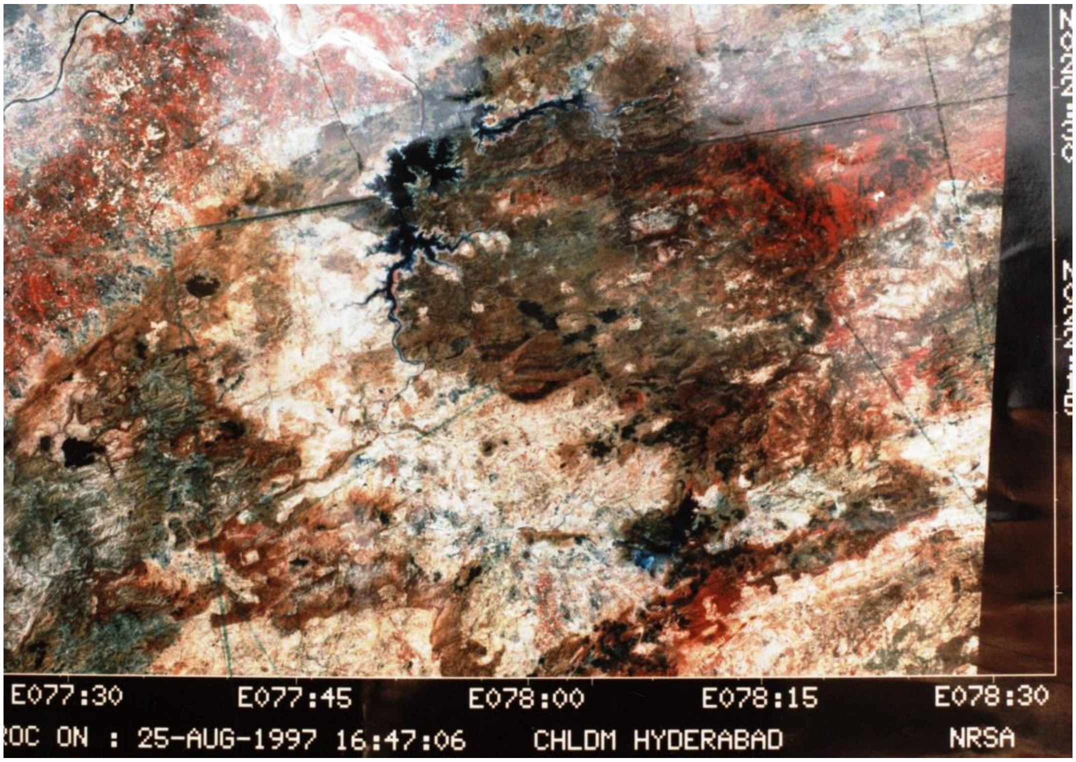


Figure 8 . PAN Stereo Images depicting the Pachmarhis

The response of the system to various kinds of input is determined by the nature of the differential equations which describe it and systems are classified in terms of the order of these equations and the typical temporal response they produce. The ultimate goal is to obtain the laws which define the system (and hence the transfer function) from completely theoretical assumptions, so that the output is defined for any input. Many attempts to define transfer functions in the natural sciences have been empirical rather than theoretical in nature. It is often argued that they are more concerned with prediction than understanding. At this point we simply wish to stress that deterministic and stochastic models meet on common ground in systems analysis.

The third major reason for adopting a deterministic quantitative approach to process response modeling is that between them, Newton and Liebnitz provided a special set of techniques for dealing with rates of change, the differential calculus. Newton's three laws of motion involve the idea of speed and rates of change, whereas Liebnitz was concerned with the formula for the gradient of the tangent to a curve. In most geomorphological applications both approaches are used interchangeably, the change of height with distance to directly analogous to the Liebnitz formulation since the ground slope is a tangent to this curve. The decrease in height of a point over time is directly Newtonian. Before using these ideas further to develop differential equations of temporal change, a brief digression into the symbols and terminology is necessary by way of revision.

A digression into elementary calculus

A basic notion in calculus is the function: a set of ordered pairs such that no two ordered pairs have the same first element. Most frequently, these order pairs are an independent variable x and a dependent variable y this idea is expressed as $y = f(x)$ where $f(x)$ formally means the

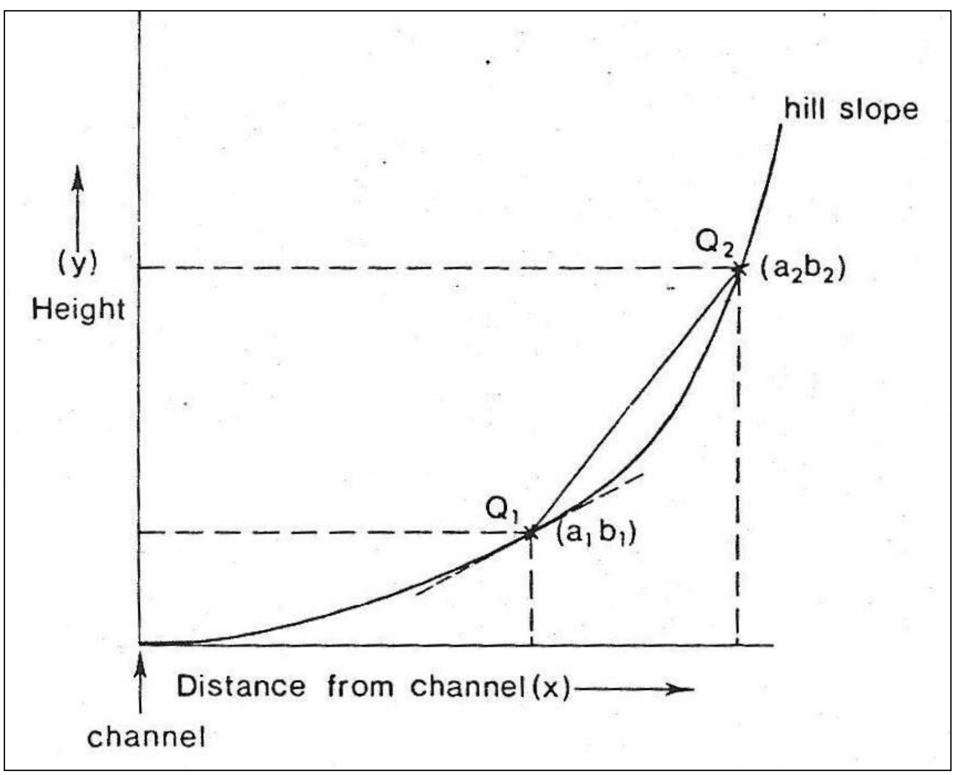


Figure. 9- Graph to show the derivation for a tangent to a curve.

value of y when x takes on a particular value. This is often generalized so that $f(x)$ refers to any equation in the variable x, and we say' y is a function of x'. The value of y may be determined by two or more variables, e.g. z and x, then $y = f(z, x)$. Simple functions can be illustrated graphically (Figure 9). We have a hill slope at the Pachmarhi and we observe two points far apart. For each point we could observe the height and distance from the channel at the foot of the slope. The gradient of the line is given by $b_2 - b_1/a_2 - a_1$ for the point Q_1 where a and b are the Cartesian coordinates of the two points. Obviously, the closer Q_2 approaches Q_1, the better the estimate of the slope at Q_1. As Q_2 approaches Q_1 instead of being a chord, it reaches the point at which it is tangent to the curve at Q_t. This is the limiting position.

It would then be meaningful to define ground slope by the tangent to that point on the curve. This procedure is empirical: we could go out and perform it in the field. Leibnitz however assumed that the slope could be defined by a function and found a method for deriving the tangent to any point on the curve. Given that height (y) is a function of distance from the channel (x) how can we derive the tangent to the slope at any point, which is the change of height for a small change in x at any value of x? This process is differentiation. In the Liebnitz notation, this tangent to the curve is the derivative and is expressed by the notation dy/dx or $f'(x)$. The expression dy/dx is simply an operation for transforming a function into its derivative. If it is a function, it too can be differentiated to obtain f, d^2y/dx^2 or the second derivative.

The first derivative of the function describing the relationship between height and distance is a function describing slope at any point. If this is further differentiated, we have the change of slope with distance, which is of course curvature. The mathematicians have developed a set of rules for differentiation and a good introduction is given in the inexpensive text by Hilton (1968). In many slope models, relatively simple relationships are assumed between height and distance, so that very difficult differential equations are avoided. Natural slopes are quite complex, but by using Taylor's theorem, the derivatives may be obtained for some polynomial of a high order (in other words a complicated function of height). Conversely, by knowing the values of the derivatives of $f(x)$ of various orders at a set of points, we could reconstruct the polynomial, which describes the slope, and hence if we wished to use it to predict the slope over distances. Such a procedure relies on the fact that the function is smooth i.e. has no relatively sharp breaks in it; this assumption is often made for convenience in mathematical slope modeling.

We have used height and distance to review the notion of the derivative, since most geomorphologists are familiar with them. The extension of first and second derivatives of the height and distance matrix into three dimensions has been especially considered by Evans (1972).

In three dimensions, the height z can be described as a function of map coordinates x and y so the function can be written as $z = f(x, y)$In this case the process of differentiation has to take into account the fact that a small variation in z cannot be a function of x alone (except in cross-section) or y alone (except in another cross-section at right angles to the first); z is partially dependent on x and partially on y. This is also true of the derivative and the procedure for differentiation is known as partial differentiation; z is differentiated with respect to x whilst y is held constant. The curled ∂ is used so that the result will clearly be distinguished from ordinary differentiation, so that the two cases are represented by $\partial z/\partial x$ and $\partial z/\partial y$ respectively. Geometrically (Figure 10) $\partial z/\partial x$ represents the slope of the curve cut from the surface $z = f(x, y)$ by the plane $y = constant$. If both these are allowed to vary, then the total differential would represent a change in the z co-ordinate of the tangent plane to the surface. As with ordinary derivatives, so with partials we may obtain second, third etc order derivatives represented by $\partial^2 z/\partial x^2$ etc. Equally, there exists a set of techniques for obtaining the partial derivatives, of any order. Finally, suppose that a variable z is a function of two other variables and these in turn are themselves functions of another variable, such as time, then z is a function of t and may be differentiated with respect to t, then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}, \quad (1)$$

For example, let z be the height of a point on a former strandline, x be the rate of isostatic recovery and y the rate of sea level change; then we could have $y = f(t)$ and with the derivatives dy/dt and dx/dt each the function of a single variable, we have two further partial derivatives if $x = f(x, y)$and so we have a general expression for dx/dtIt is no surprise to find that the expression for the ordinary derivative occurs relatively rarely in geomorphic model-building. Invariably, we are examining situations in which some variables are held constant, not least of which is usually the horizontal spatial co-ordinate, for example in slope studies.

Differential equations

Consider a hill slope in which denudation is proportional to the height of a point under consideration above a certain base level. Scheidegger (1961) suggests this elementary model to introduce more complex models (given the assumption that precipitation increases with height, this is not absurd). The height loss considered will be measured vertically (Figure 11) so that the model can be expressed by saying by

$$\frac{\partial y}{\partial t} = -y \qquad y = \text{height}, t = \text{time}, \quad (2)$$

This is a differential equation of the first order because the equation contains a derivative and the derivative is the first derivative of some function. A solution of a differential equation is that expression for the dependent variable which does not involve any of its derivatives and which, when substituted into the given equation, reduces it to an identity. The solution to this particular differential equation is given by

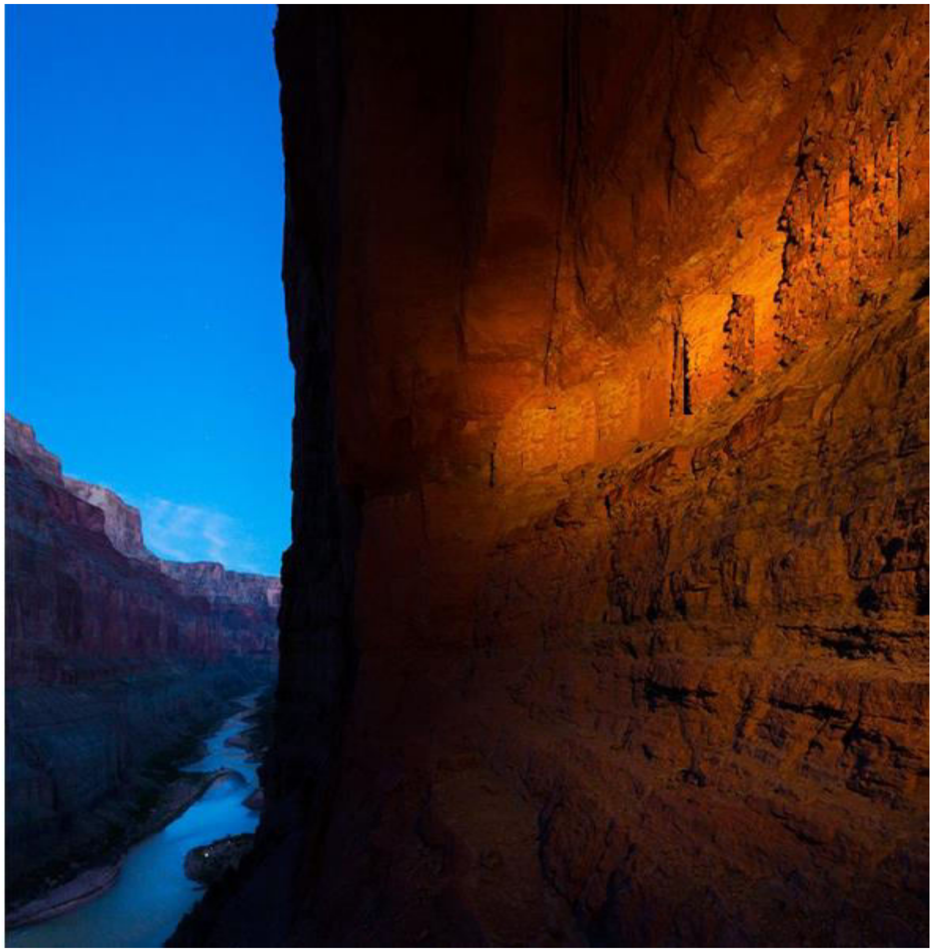
$$y = f_0(x)e^{-t}, \quad (3)$$

which can be evaluated for given values of x after time t If we further assume that some constant relates height and rate of removal, we shall have $y = 2x.e^{-ct}$ where c is the constant.
Reviewing this example, we note the following steps:

1. Conversion of a verbal statement into a differential equation;
2. Solution of the differential equation to obtain a derivative-free function of a most general character;
3. Specification of the initial conditions;
where $y_0 = f_0(x)$ describes the original land surface which is subject to change. For example, suppose $= 2x$, describes the original landform then the solution is given by

$$y = 2x.e^{-t}, \quad (4)$$

4. Substitution of any real parameter values ($e.g. a\ value\ for\ the\ constant, c$) into the solution, and
5. Examination of the results.



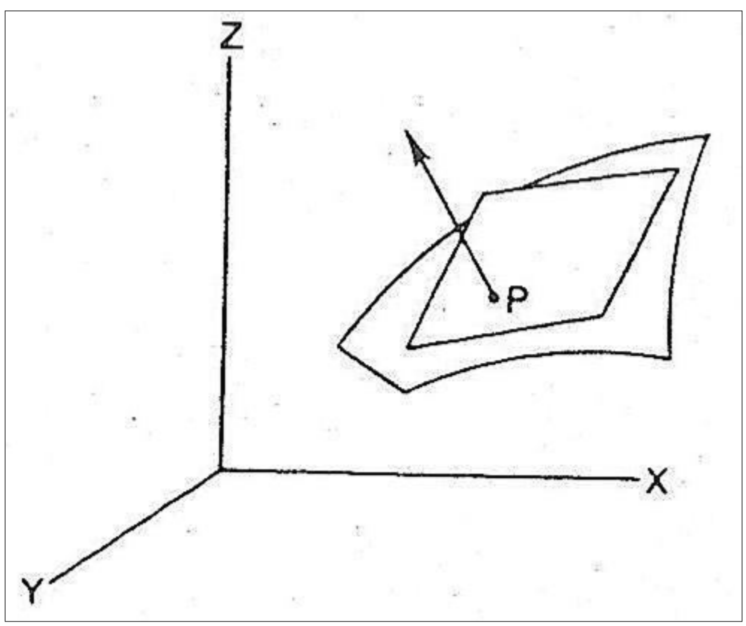


Figure. 10- Tangential plane to a surface at a point P. The arrow indicates the normal to the plane.

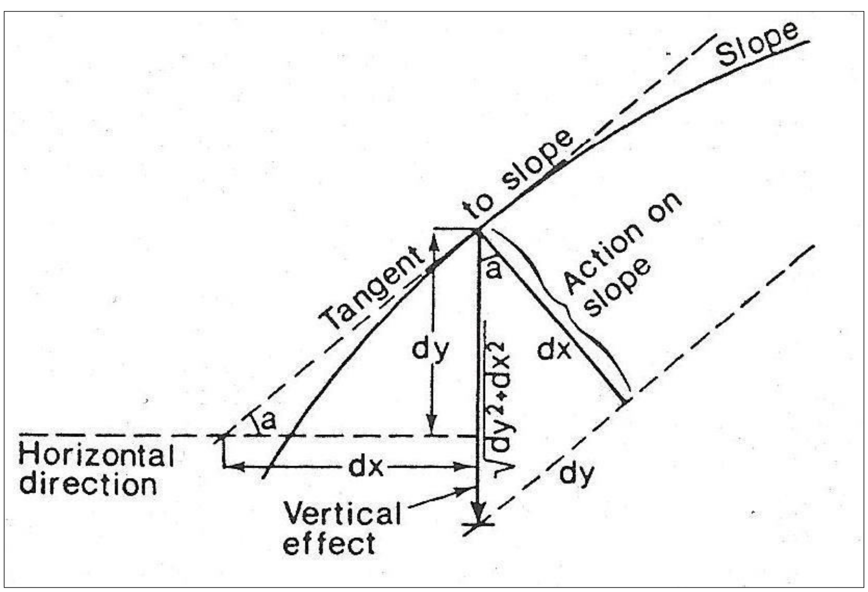


Figure. 11- Terminology for the derivation of slope evolution equations (after Scheidegger 1971).

It is quite important, in reading literature on quantitative deterministic models, to be able to identify the separate steps. If this is done, the reader will put himself in a more usefully critical frame of mind. Differential equations and their solutions owe their importance to the fact that there is a clear correspondence between them and the situation they represent. They usually provide a clear and simply expressed model of a somewhat complex physical situation. Step 2 usually presents most difficulty. Most of the important differential equations of mathematical physics have been derived from the process of separation of variables. Their solutions have been given special names such as Bessel, Legendre and Mathieu functions and their properties described in reference books of mathematical functions. There are about 2,000 functions with known solutions in all. One consequence of this is the tendency of some workers to cast the problem and its differential equations in a form for which the solutions have been developed elsewhere. In three examples later, we shall show how this applies to slope studies and the diffusion equations, glacier flow and characteristics, and rejuvenation and perturbation theory.

The differential equation described above is of the simplest type. A somewhat more complex model is given by

$$\partial y/\partial t = a \ . \ \partial^2 y/\partial x^2, \tag{5}$$

which is a second order differential equation (Culling 1960) in which the change of $height$ (y) with $time$ (t) is a function of the local curvature multiplied by a $constant$ (a). The solution to this equation is known from heat-diffusion Problems in physics and is given by and shown in (Figure 12)Finally, Hirano (1968) suggested the model

$$\frac{\partial z}{\partial t} = a\frac{\partial^2 z}{\partial x^2} - b\frac{\partial z}{\partial x} - c, \quad \text{z}, \tag{6}$$

($where\ a, b\ and\ c\ are\ \text{'}erosional\ constants\text{'}$) which comprises a combination of the earlier models. The procedure of finding a solution is as outlined above. Once a general solution is found, then the parameters may be changed; again the differential equation is of second order. The other character which the three models have in common is that all are linear models. This is discernible by the fact that none of them involves powers or products of the dependent variable y or its derivatives. Linear differential equations are the only ones for which a complete analytical theory exists and for which general analytical solutions can be obtained. Most procedures for solution of non-linear equations consist of 'linearizing' the equation and then using one of the standard techniques for obtaining solutions.

Scheidegger (1970) points out that the above models should account for the lowering of slopes normal to the surface rather than vertical. The geometry involved leads to non-linear equations which have to be solved by different techniques. For example, where $\partial y/\partial x$ is made a function of height, the corresponding equation is

$$\frac{\partial y}{\partial t} = y\left[1 + \left(\frac{\partial y^2}{\partial x}\right)\right]^{\frac{1}{2}}, \tag{7}$$

which is non-linear because the derivative $\frac{\partial y}{\partial x}$is squared. In obtaining solutions the differential equation is converted into a difference equation and this is solved on the computer. The results for this model are given in Scheidegger (1970).

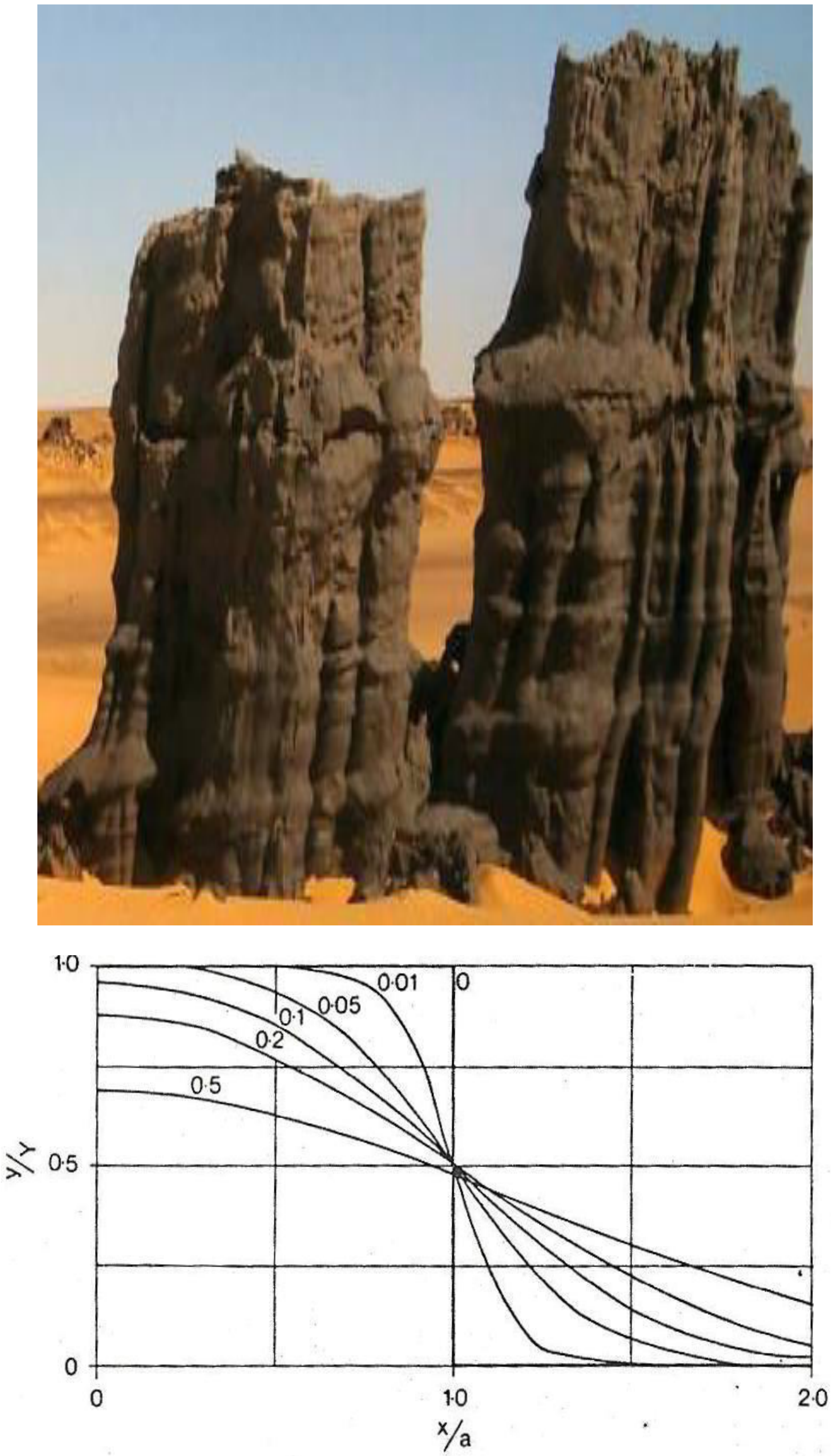
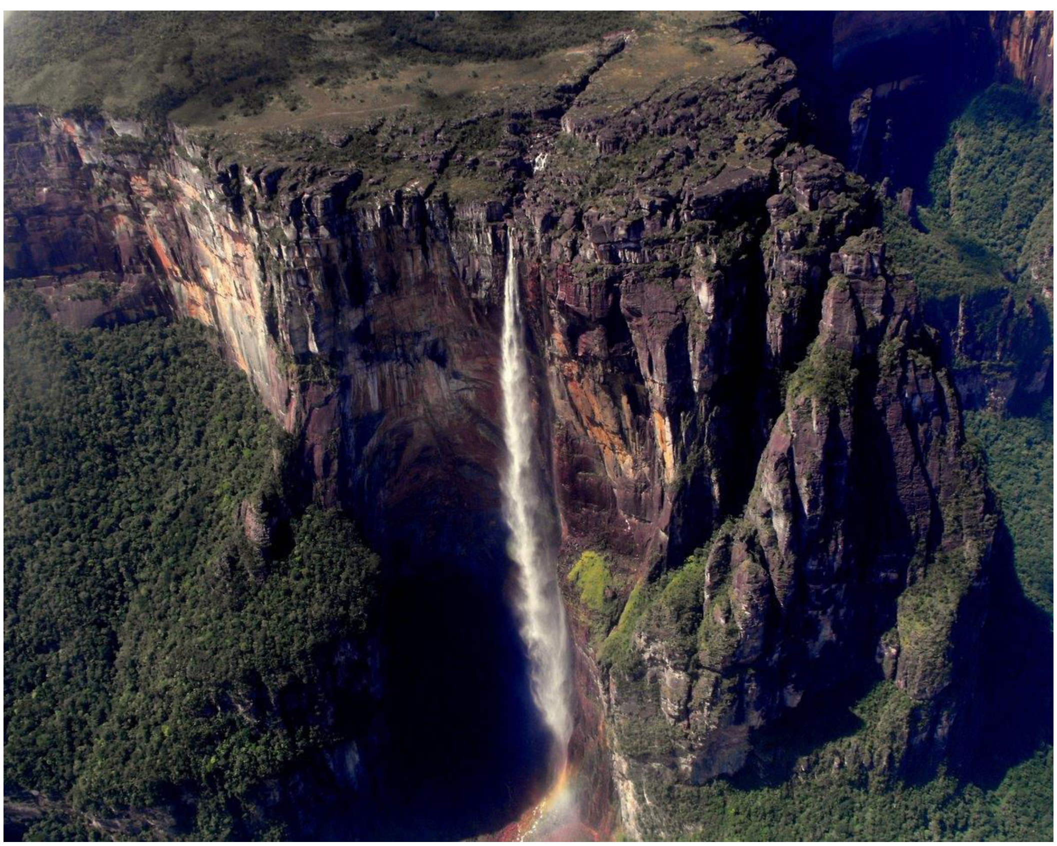


Figure.12- Decay of a vertical cliff, where the change in height with time is a function of curvature (after Culling 1960).

One may ask what happens at, the crest and at the stream Channel. This raises the last general point concerning differential equations, that of boundary conditions. The above solutions were general in that (except in one case) the initial conditions and parameters were not specified. Obviously, in interpretation we have to have real, particular values for the solutions. In addition to providing the initial conditions and any constants we have to specify the conditions at the boundaries, i.e. the values which must obtain at two or more values of the independent variable. For example, we could assert that in the above situation a boundary condition is that $y = 0$ at $x = 0$, i.e. base-level is at the foot of the slope. Another one, very familiar to geomorphologists, is that velocity is zero at the bed of the stream i.e. when *y,* the distance above the bed is zero. Obviously, the boundary conditions may themselves be time dependent; thus, the value of y at $x = 0$, the height of the stream channel, could be lowering through time, for example in a simple linear fashion (Culling 1963). Where boundary In a zero order system, using the terminology of Grodins (1963), the response or output from a process (forcing function, input) is independent of time, so that a time derivative is absent from the equation describing the system. Thus a zero order system simply multiplies the input by the transfer function (which in this case is simple gain) but does not change the timing between input and output which is one of instantaneous response.

Equations of systems

We have already outlined the types of differential equations which exist, partial and ordinary, linear and non-linear, and first, second, third order. Linear systems without feedback may be defined in terms of the order of the differential equation which describes the system's operation with respect to time. level determined by the hydraulic conductivity and infiltration capacity rate of the soil (Figure 13).



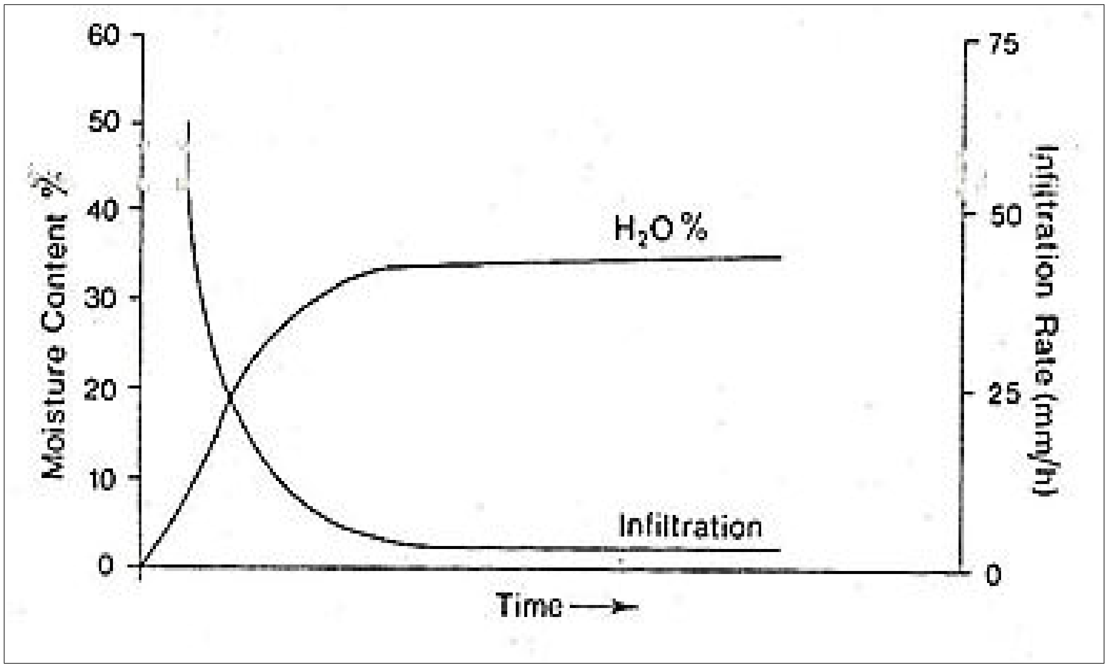


Figure.13- Hypothetical infiltration response to precipitation.

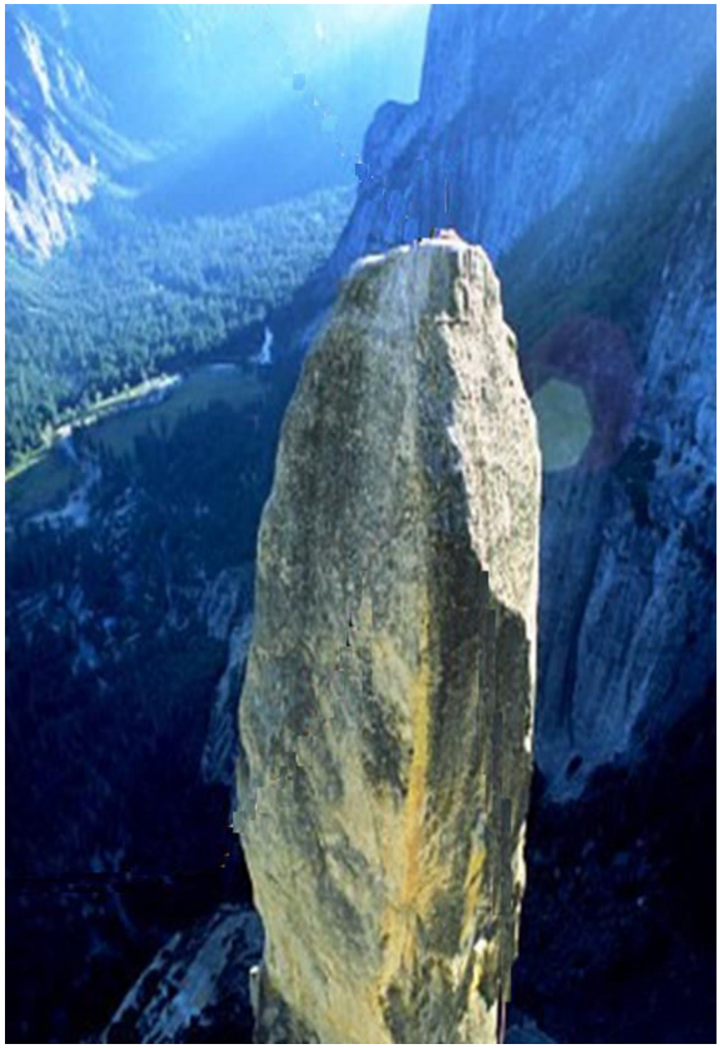
In a second order linear system, the derivative in time is second order in at least one term. Typically, it should take the form

$$b_1 \frac{\partial^2 x}{\partial t^2} + b_2 \frac{\partial x}{\partial t} + b_3 x = c, \quad (8)$$

The existence of the second order term, taken together with the first-order term, implies that the response will be damped to a steady state again, but that with particular values of the coefficients b_1, and b_3 the system may (a) reach equilibrium like a first order system, or (b) reach the steady state level by a series of damped oscillations. An example of this is shown in (Figure .14) which illustrates the damping of groundwater inflow to a stream caused by passage of a flood wave. Another important case is the passage of a surface temperature wave ($positive\ or\ negative$) into the ground. This takes the general form of a damped sine wave curve with depth. So far little application has been made of second order models in geomorphology, though there are several areas in which they might be expected. One is in the study of sea-level fluctuations, where damping of climatic oscillations and the glacier response seems to have been important. Again, the pattern of isostatic rebound might imply the existence of a double energy storage phenomenon which is characteristic of these second order systems.

Equations of continuity and diffusion

One important component of equations describing the behavior of continuous matter is the requirement that all mass is accounted for. Put at its crudest level, this could be called the 'what-goes-in-must-come-out' equation. Together with an equation of motion, an equation of state, a kinematic condition and the appropriate initial and boundary conditions it provides a complete description of the behavior. In all geomorphological situations mass is being moved from one position to another, whether soil, water, solid rock, channel debris, or solutes mixing in a stream. The conservation of mass is so absolutely fundamental that it forms the core of most physical models. It is sometimes called the equation of continuity and has two expressions, one representing steady flow of an incompressible fluid and known as the Laplace equation; the other



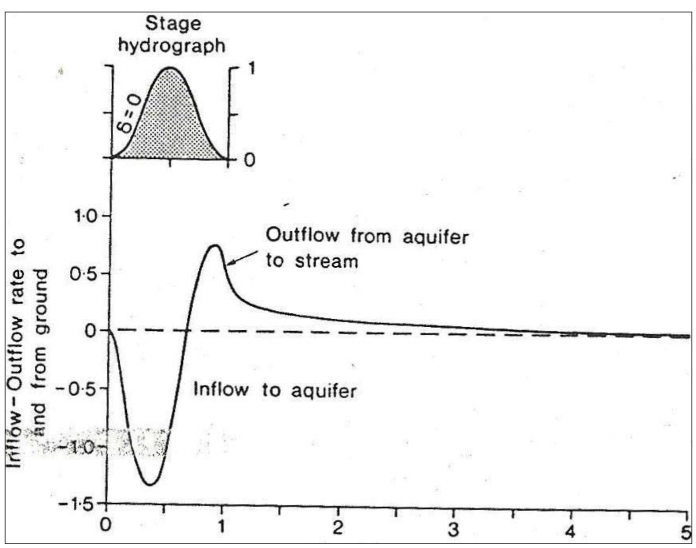


Figure.14 . The damping of groundwater inflow and outflow to a steady state condition caused by the passage of a flood wave in a stream channel (after H. H.Cooper, Jr, and M. I. Rorabaugh 1963).

a time-dependent flow, occurring before steady flow is reached and generally known as deterministic diffusion. The last term is to differentiate it from probabilistic diffusion. In the latter models we expect to find a partial derivative with respect to t, and indeed this is the case.

If we imagine a simple cell, whose three axes are $\Delta x, \Delta y$ and Δz (Figure 15) we can use geometry and some simple symbols to obtain the steady-flow model. The mass flow into the left-hand side of the cube is given by $M_L = \Delta y \Delta z\, p_L v_L$where v is the velocity of flow through that face, $\Delta y \Delta z$ is the area of the face and p is the density. A similar expression can be obtained for the right hand face and the difference between them given by:

$$\Delta M_x = \Delta y \Delta z p_R v_R - \Delta y \Delta z p_L\, v_L, \tag{9}$$

which, taking out the common elements and letting Δagain mean 'difference'.we have:

$$\Delta M_X = \Delta(pv)x \Delta y \Delta z, \tag{10}$$

This is also true in the y and z directions and the notation becomes:

$$\Delta M_y = \Delta(pv)y \Delta x \Delta z, \tag{11}$$

$$\Delta M_z = \Delta(pv)z \Delta x \Delta y, \tag{12}$$

Now the conservation equation says that:

$$input — output = accumulation,$$

In this case accumulation is represented by the change in mass of the fluid element, and hence of average densities p_1 and p_2 over a short period of time. The equation then is

$$input\text{–}output = \left(\Delta M_x + \Delta M_y + \Delta M_z\right)\Delta t, \tag{13}$$

that is, the change in flow across the faces in a unit of time, and accumulation $= (p_1 - p_2)\Delta x \Delta y \Delta z$. Thus letting$\Delta p_1 = p_1 - p_2$ .we have the unpleasant looking equation:

$$[\Delta(pv)x \Delta y \Delta z + \Delta(pv)y \Delta x \Delta z + \Delta(pv)z \Delta x \Delta y]\Delta_t = \Delta p_t \Delta x \Delta y \Delta z, \tag{14}$$

and if we divide through by $\Delta x \Delta y \Delta z \Delta t$ we are left with:

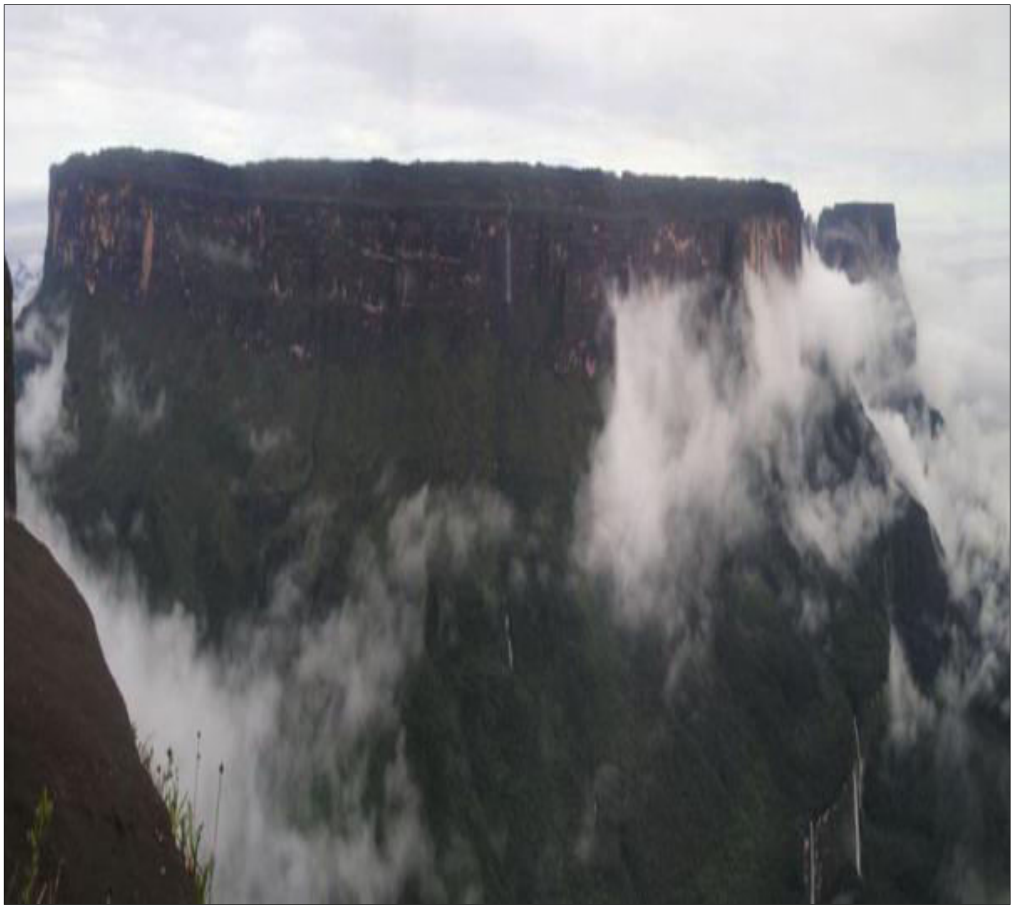
$$\frac{\Delta v_x}{\Delta x} + \frac{\Delta(pv)y}{\Delta y} + \frac{\Delta(pv)z}{\Delta z} = \frac{\Delta pt}{\Delta t}, \tag{15}$$

If the fluid has constant density, then p is constant and:

$$\frac{\Delta v_x}{\Delta x} + \frac{\Delta v_y}{\Delta y} + \frac{\Delta v_z}{\Delta z} = 0, \tag{16}$$

as the values are considered continuous and very small, this relationship can be expressed by the partial differential equation:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0, \tag{17}$$



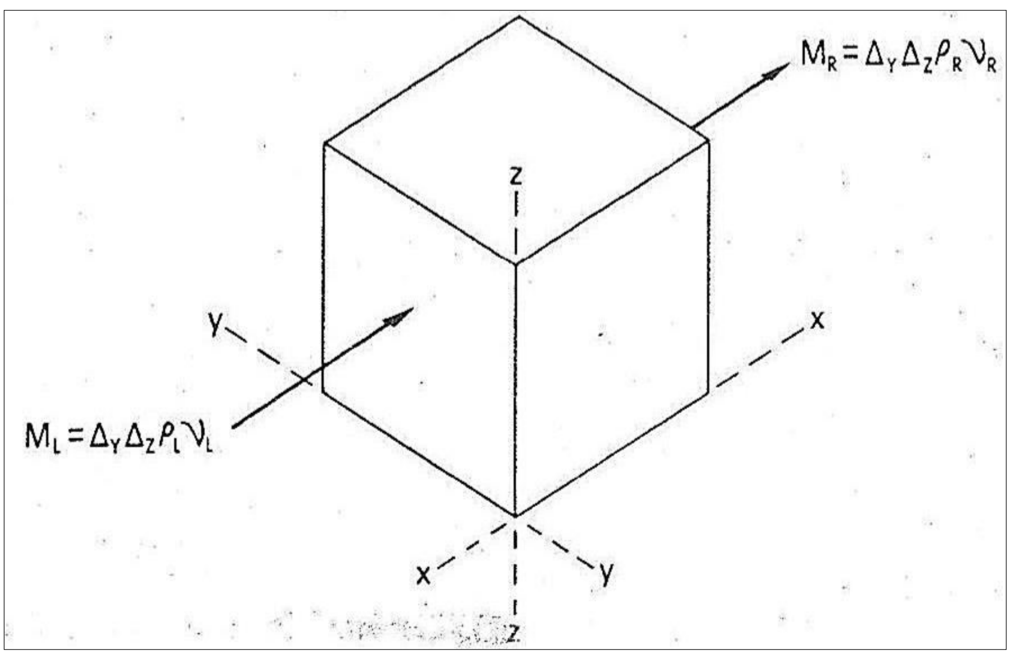


Figure.15- Notation for development of steady state diffusion model.

Notice that there is no derivative with respect to t, as expected. Now velocity is a function of the change in velocity potential Φ in any particular direction, e.g. $v_x = \partial\Phi/\partial x$ and we have (substituting in the above equation) the second-order differential linear equation:

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0, \tag{18}$$

which is the Laplace equation. The expression for the summed second derivative of a variable in three dimensions, i.e. $\partial/\partial x^2 + \partial/\partial x^2$ is given by ∇^2 called the Laplacian operator, so the above equation can be represented by

$$\nabla^2\Phi = 0, \tag{19}$$

Various analytical, graphical and experimental techniques are used for showing this basic equation. For a unit volume, the expression used above, that velocity is proportional to potential gradient

$$v_x = -K\frac{\partial\Phi}{\partial x}, \tag{20}$$

is used for flow in soil, where $\Phi = \ hydraulic\,potential$. With steady flow in a homogeneous, isotropic medium the flow can then be described by Darcy's Law, in which K is the coefficient of diffusion.

$$v_x = -K\frac{\partial h}{\partial x}, \tag{21}$$

These conditions are relatively rarely encountered in natural soils, but the formulation is important because it allows simple models to be built and forms a bridge to the diffusion models, which are also based on the contineuity model, $input - output \ = \ accumulation$. Here, the basic assumption made earlier, that flow is steady and time independent, is relaxed. The concentration of mass in the cube is assumed to vary through time, and of course this change of mass represents accumulation or loss. Assuming again matter which is incompressible (density remains constant) then the change in concentration depends on the mass-flow into and out of the cell through its various faces. If J_x is the net mass-flow in the x direction, then the accumulation is the sum of the net mass-flow through all the faces, expressed as:

$$\frac{\partial c}{\partial t} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}, \tag{22}$$

$where\ c = concentration\,accumulation = input\text{—}output$, Now, $J_x = -K\partial c/\partial x$, where $\partial c/\partial x$ the concentration gradient and K is is a diffusion coefficient in the x direction. Similar expressions can be obtained for the other directions and if we assume that K, the diffusion coefficient, is constant in all the directions, then substituting in the previous equation, the result is:

$$\frac{\partial c}{\partial t} = -K\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}\right), \tag{23}$$

$$\frac{\partial c}{\partial t} = -K\nabla^2 c, \quad (24)$$

This important, fundamental differential equation is called Fick's second law of diffusion. Together with the condition of continuity from which it is derived, it forms an important core of-mathematical geomorphological theory. As the continuity condition has different formulations, so too does the diffusion equation and its solutions. However the basic form remains essentially that described above. Sometimes the equations are simplified rather than made more complex by the fact that they may be taken in only one or two directions; the initial and boundary conditions still have to be specified. Once again it is important to note that while the formation of the problem into differential equations is the primary field of geomorphological interest, solution of the equations, subject to various conditions, is a substantial task.

$$\frac{V_R}{V_D} = (1 - C)$$

$where\ c\ is\ a\ constant\ and\ VR\ is\ volume\ of\ bedrock\ and\ KD\ is\ the\ volume\ of\ debris$.

These volumes are expressed in differential terms to obtain an equation for y $(height)$ in terms of x $(distance)$ and the initial slope of the cliff-face β .The general solution can be made particular by inserting various values of a and c and some solutions are shown in (Figure 16)

Another continuity formulation for the conservation of ice-mass in an infinitely wide glacier is given by:

$$\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} = b, \quad (25)$$

which is the fundamental starting point for study of the motion of a glacier. In this equation the net mass balance (b) is equal to change in the depth of ice flow $(h) +$ the change in ice discharge q, while x is the coordinate direction. An almost identical expression:

$$\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = i - f = i_0, \quad (26)$$

(Eagleson 1970, p. 332) may be used to express continuity in overland flow. If $i - f = i_0$,$where\ i = point\ rainfall\ intensity$, $f = infiltration\ rate$ and i_0 rainfall excess intensity, then this is equated with the change in flow depth in channel $(y) +$ the change in overland flow discharge per unit width of channel (q).A third example of the formulation of a continuity expression is from Kirkby's work on hill slope processes (Carson and Kirkby 1972). Kirby's expression, for which the terms are shown in (Figure 9) is : $debris\ transport\ in\ -$
$debris\ transport\ out\ over\ a\ unit\ length\ of\ slope\ profile\ -$
$increase\ of\ soil\ thickness\ due\ to\ weathering\ and\ addition\ =$
$decrease\ in\ elevation\ of\ land\ surface.$

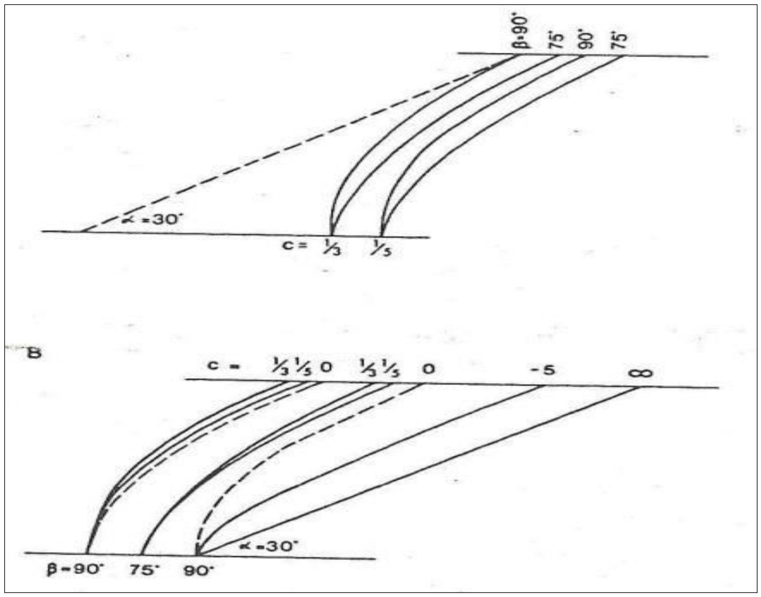


Figure.16- Cliff recession under the models of Bakker and Le Heux (1952) (a) for various initial Conditions of the cliff face β given slope angle for the screen (a) and (b) for condition of volumetric change (c).

The difference in debris transport (in and out) is expressed as $\partial s/\partial x$the change in soil thickness due to weathering is $(\mu - 1)W$, where μ is a constant similar to Lehmann's c (Lehmann 1933) and W is the weathering rate; the ground loss in time is given by $-\partial y/\partial t$ (being negative to indicate ground loss). The full expression is then given by:

$$\frac{\partial s}{\partial x} - (\mu - 1).W = -\frac{\partial y}{\partial t}, \tag{27}$$

A second continuity equation in Kirkby's work relates to change in soil thickness, thus in differential terms:

$$\frac{\partial z}{\partial t} = \frac{\partial y}{\partial t} + W = \mu.W - \frac{\partial s}{\partial x}, \tag{28}$$

$where\ \partial z/\partial t\ is\ increase\ in\ soil\ thickness$

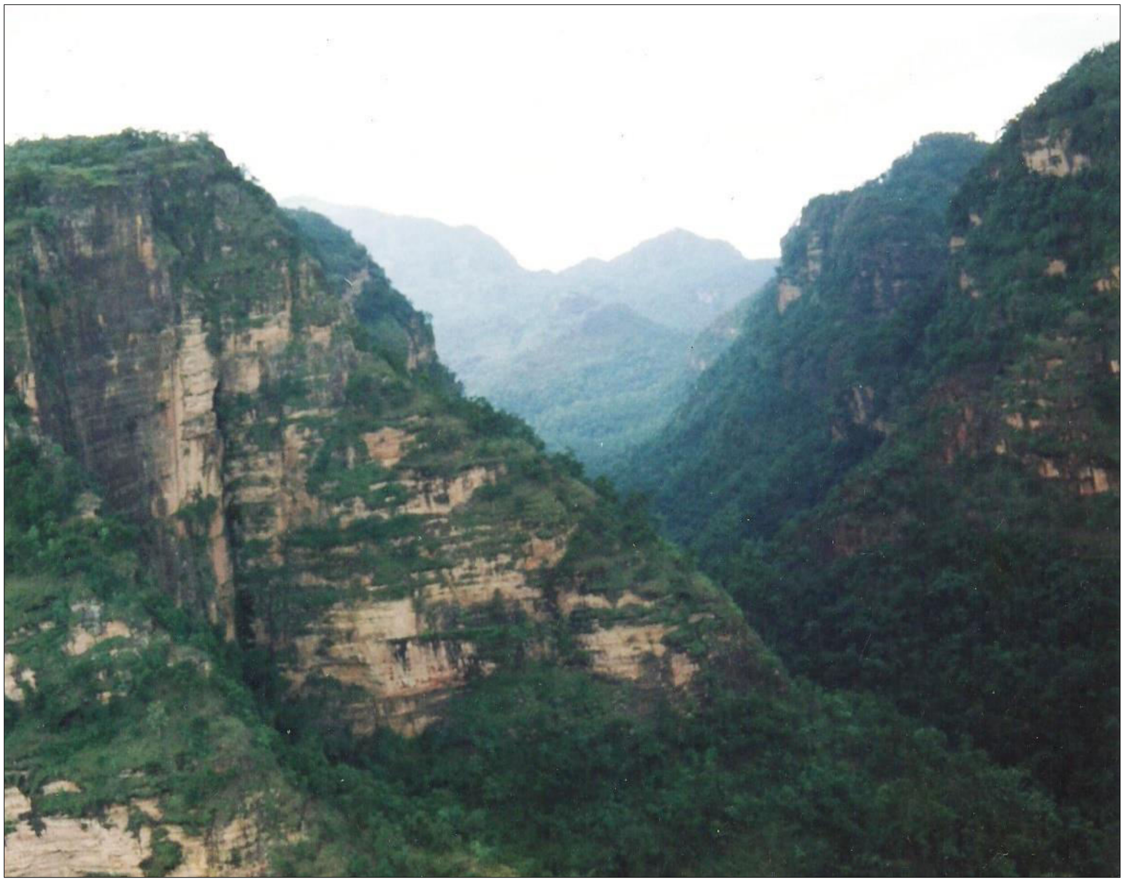
The diffusion equations were first developed for work on heat conduction, and it is mainly through soil temperature and glaciology that they make an appearance in geomorphology. On a glacier the surface cold wave is transmitted down into the glacier. The annual 'wave' may be treated in this fashion, and it can be described by the expression:

$$T_t = T_s \sin Wt\ (boundary\ condition, y = 0), \tag{29}$$

e surface, T_s the amplitude of the surface temperature wave and

$$\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial y^2},$$

where y is depth in the ice and K the thermal diffusivity coefficient. Thus if a illusion can be obtained, the result will give the temperature at depth y and me t, expressed as $T(y, t)$ and given by the equation



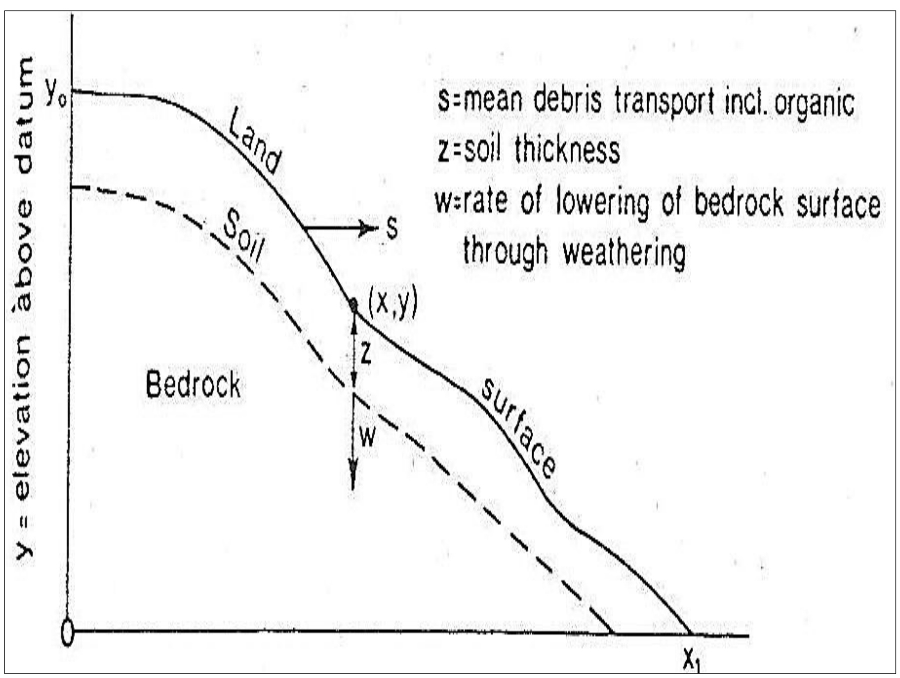
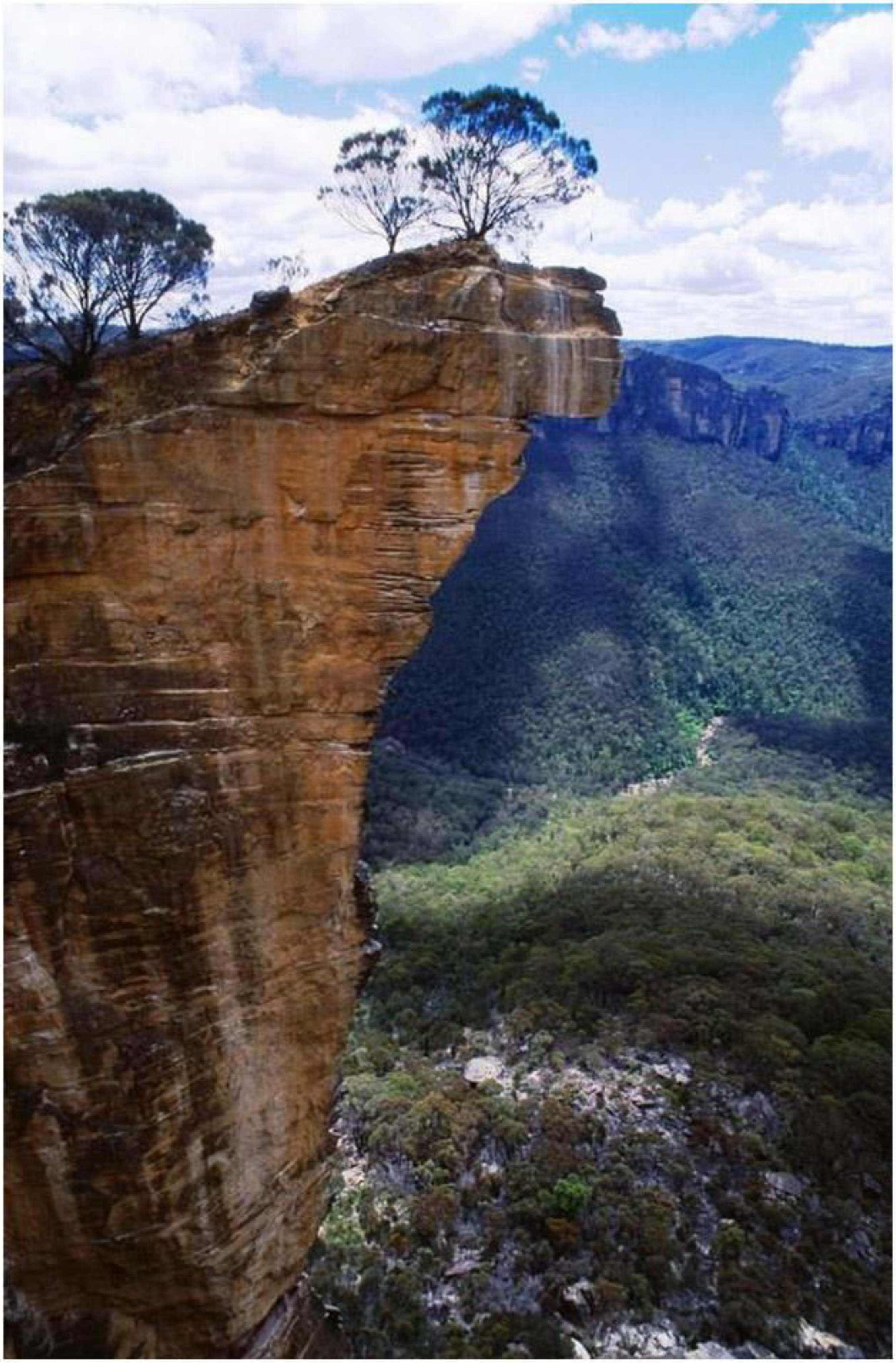
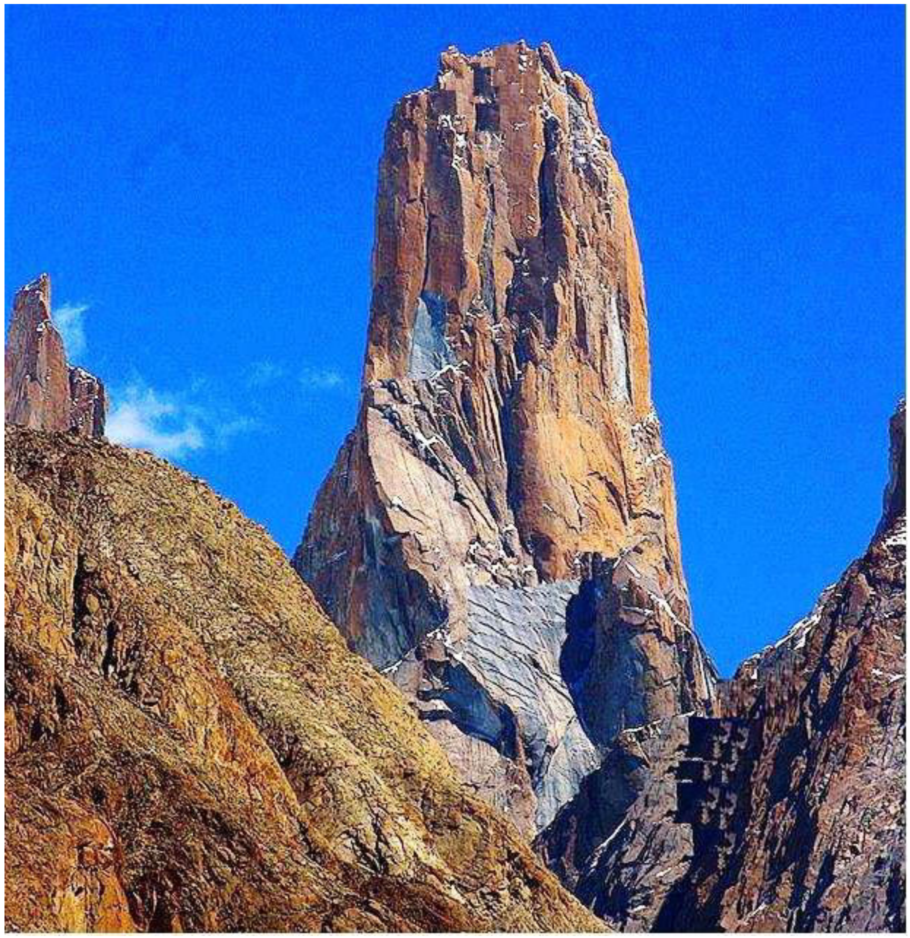


Figure 17. Terminology for formulation of the continuity equation (after Carson and Kirkby 1972).





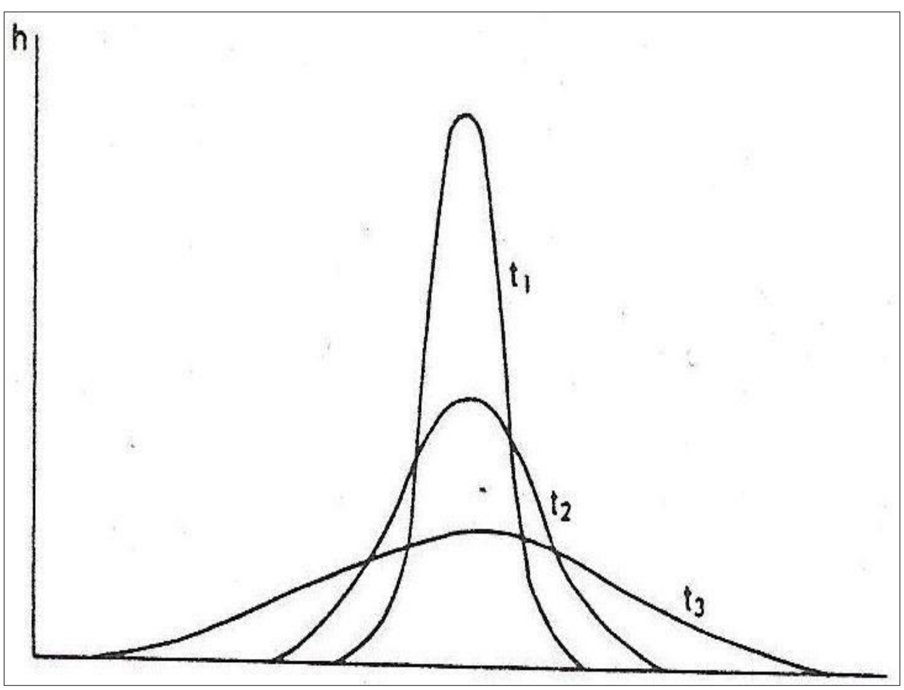


Figure 19 . ecay of a mountain under the assumptions of the diffusion model for three successive time periods (after Scheidegger 1970).

$$= \frac{1}{(4\pi Dt)^{½}} \exp\left(-\frac{x^2}{4Dt}\right), \tag{30}$$

where T_t $is\ temperature$, $t\ time$, W_t , the frequency of temperature change at (Figure19) Which when plotted for three values of time yields the results shown in (Figure 19)As was mentioned earlier, the continuity equations represent only one of the elements required to describe the behavior of matter. The other essential components in most dynamic time models are the equations of motion. These take on an even wider variety of forms, and will not be discussed at length here. Instead, three interesting and important models for change through time will be used to demonstrate the wider aspects and implications, as well as procedures, for this type of modeling.

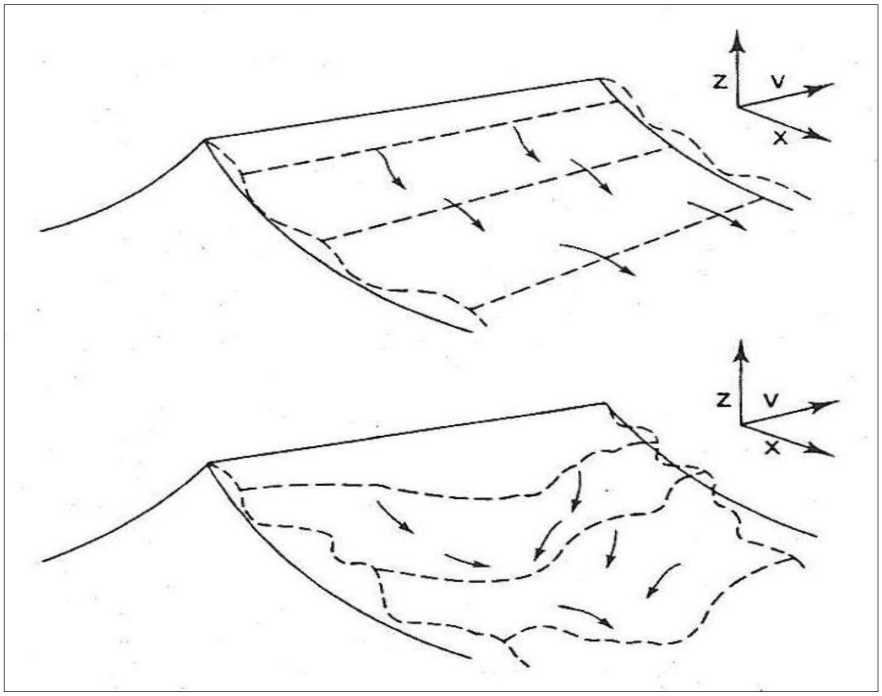


Figure.20- The effect of perturbations on surfaces. The upper diagram shows the effect on a smooth slope with perturbations in two dimensions parallel to the $x - axis$. The lower diagram shows perturbations in three dimensions. See text for more detailed explanation (after Smith and Bretherton 1972).

Stability analysis

The last example discusses some recent, experimental work by Smith and Bretherton (1972). The problem is to obtain solutions to equations describing the effects of perturbations on (1) a surface which is initially fairly smooth, Fixed depths. Notice in (b) how the positive irement in temperature at $Time = 0$ reaches $d = 5$ after about 3 months and (2) a $V - shaped$ channel system. The procedure is to set up a model using basic continuity laws and equations of motion. This model, represented as usual by partial differentiation equations, is then subject to a perturbation. In other words, in the drainage basin, which is everywhere in steady state, the surface is instantaneously modified by a small amount. If this small perturbation gradually disappears when the model is re-started, then the basin is said to be stable. If, on the other hand, after re-starting the model the perturbation (i.e. depression or knick point) begins to grow, then the basin is unstable. This technique unfortunately applies only to small perturbations, so that if the system is unstable, the assumptions of the technique prohibit us from following through the evolution of the system for any length of time. In their first experiment, Smith and Bretherton consider perturbations in two dimensions only, parallel to the $x - axis$ (Figure 20).

In this case, perturbations are removed because an increase in slope causes an increase in sediment transport whereas a decrease in slope results in decrease in transport. (Figure 20) One implication is that knick points will always be removed by migration 'up-gradient', an observation which is supported by flume work. The second experiment relates to perturbations in three dimensions. Given their transport law, it is concluded that with a constant form surface which is elsewhere concave, there can be no stable channel system on the surface; if it is straight or convex there is no instability and channels cannot develop from small perturbations. With a landscape combining the two elements, one part (the convex) would inhibit channel development (negative feedback) whereas in another area it would be unchecked (positive feedback).

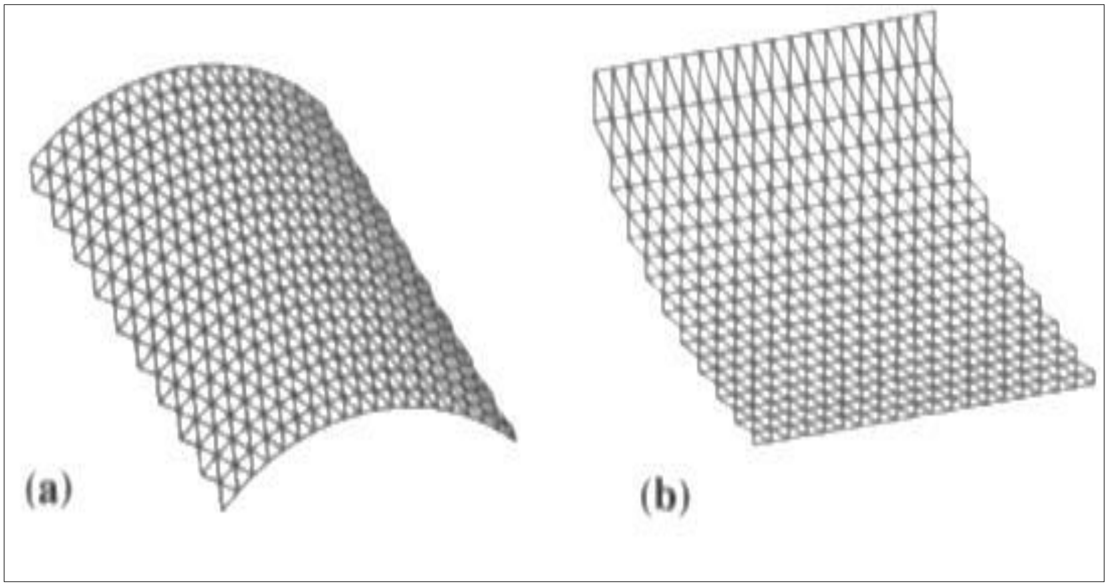


Figure. 21 Fig a and b are showing the two steady-state surface of Pachmarhi. These figure are prepared by Lagurerres method.

This idea of stability and instability through space is quite important, and conforms to the idea of a transient spatial behavior comparable to transients in time. It is in the linking of spatial and temporal components that the full benefits of deterministic mathematical modeling will be reaped. At the moment, as this brief review has shown, the Difficulties of obtaining analytical solutions for models which are cast in several dimensions and especially those which are non-linear are very considerable and, as yet, the amount of three-dimensional analytical modeling is very modest. Some of these problems may be overcome by simulation but at present, despite these problems, analytical modeling is the most powerful tool of theoretical geomorphology

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