Perturbation of thermal equilibrium in the Satpura basin

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ABSTRACT - The Pachmarhis summits, ridges and plateau, indeed all protuberant land forms , are the features actually formed due toTempe

rature variations due to the formation of the Satpura basin The Perturb bation of the thermal equilibrium, produced by subsidence and sedimentation in the earth's outer layers is investigated by means of two models. In one the heat reaching the surface is assumed to come from the deep interior, in the other, to be generated in the crust. The models consist of three horizontal layers, the top one representing the sedimentary layers, the middle one representing the crystalline part of the continental crust, and the lowest one representing the plastic substratum. It is assumed that at a particular instant the thickness of the top layer is increased. A theory is developed to determine the subsequent temperature variations. Also, asymptotic values are found for large times. A finite subsidence velocity introduces an uncertainty in the time of origin of the rapid-subsidence model. For events which occur long after the subsidence, the instantaneous assumption is adequate. Moreover, it is adequate without this restriction in the lower part of the crust and in the substratum. For large times, the deviation from the final steady state is a linear function of the depth. When the heat comes from the interior, the temperature deviation from the final steady state varies as f^3l^2, whereas when the heat is generated in the crust, it varies as $t^{-1/2}$. The temperature variations are evaluated for subsidence of 6 and13 $kilometer$. The sedimentary layer is assumed to be 2 $kilo\ meter$ thick initially, and the crystalline crust is assumed to be 30 kilometer thick. The time variation of the different components of the temperature variations gives an insight into the propagation of thermal disturbances in the earth. Thermal adjustment requires millions of years. After subsidence, the temperature increases rapidly during the first $20\ million\ years$ or so. Thereafter, the rate of increase is much smaller. The increase is much more rapid in the sediments than in the lower part of the crust. At the base of the crust, the increase is about $22° - 23\ °C\ per\ kilometer$ of subsidence, if the thermal conductivity is $0.006\ cgs$. However, if the conductivity decrease with temperature in the range of crustal temperatures was taken into consideration, much larger increases would be obtained. The times required for thermal adjustment are large enough to be significant in certain geological processes. The temperature increase after subsidence should affect the rate of lithification of sediments and the strength of the crust. Both the stresses and the temperature of a certain portion of sediment increase with subsidence. At depths greater than a few hundred feet, the confining stress on the grain matrix corresponds, practically without any time lag, to the current depth of burial. On the other hand, the adjustment of the temperature lags appreciably. Thus, lithification may be incomplete at certain depths because the temperature is still far below its final value. Lithification is likely to be completed within the first $20\ million\ years$ after the subsidence. During the folding stage of a geosyncline, the shearing rate of the crustal rocks increases appreciably. The crustal "Solid Viscosity" may decrease markedly when the temperature reaches the value it has at the base of an undisturbed crust. Thus, the crustal strength decreases relatively quickly up to about 20 million years after the subsidence and thereafter much more slowly. Because of this decrease, the folding stage of a geosyncline would occur $15 - 20\ million\ years$ after the subsidence. If heat comes from the deep interior, there is, for a certain time, a zone of cooling in the substratum. The corresponding increase in the solid viscosity of the substratum may effectively lock, for a certain time, the subsequent uplift of the crust.

Introduction

The Satpura Sediments are distributed in a trough like depression. In most cases sedimentation keeps pace with subsidence, so that the trough is constantly filled. Flow of the plastic substratum permits the subsidence of the crust. The subsidence of the crust and the piling up of sediments in the trough upset the thermal equilibrium. During the subsidence, the isotherms are displaced downward; when subsidence ceases, they move upward, thus gradually approaching their new steady-state positions. Hence, the formation of a basin causes temperature variations in the outer layers of the earth. This paper studies these temperature variations.

Because most rock properties depend somewhat on the temperature, the temperature variations obtained here should interest investigators in other fields of geology. Mechanical properties such as strength and plasticity depend on the temperature. The evolution of basins depends on these properties. Some sediment is buried to great depths by subsidence, and their temperature and confining pressure increase. Since temperature and pressure are important for lithification, 86a knowledge of the temperature variations should be of interest in dating and studying transformation of sediments into indurated rocks.

The temperature variations depend on the initial or normal temperature distribution in the outer layers. There is, however, some uncertainty about this. All the heat reaching the surface could be accounted for by radioactive material in the upper $30\,kilometer$ of the crust. Many geologists assume, however, that some heat comes from the deep interior and represents a gradual cooling of a once-hot earth. This paper considers both these assumptions about the source of the heat.

Several studies have been made of perturbations of the temperature distribution in the earth's crust. Jeffreys (1931) investigated the thermal effect of adding to the crust a thick cover of sediments, but he determined only the temperature at the base of the sediments and the surface gradient. The sedimentary cover was regarded as added instantaneously. He assumed radioactive sources of heat were present in the crust and absent in the sediments. Furthermore, he assumed an initial distribution of temperature having a constant gradient.

If the radioactivity were uniformly distributed, the equilibrium-temperature distribution would be parabolic in the zone of the radioactive sources. Since Jeffreys was interested only in the temperature at the base of the sediments and in the gradients at the surface, the resulting discrepancy is not great. However, determination of the temperature variations throughout the crust and substratum depends on the initial temperature distribution. Therefore, the temperature variations are discussed under the two hypotheses about the nature of the heat. This paper also considers the effect of the distortion of the plastic substratum, which is important for the determination of the temperature variation in the lower part of the crust. Jeffreys (1938) and Bullard (1938) studied the effect of topographic inequalities on the geothermal gradients at the surface. Except in very mountainous country, this effect is only a few per cent of the normal values. Coster (1947) examined the effect of anticlinal structure using an electrical model. Benfield (1949) and Birch (1950) studied the simultaneous effect of uplift and erosion.

Notation

The following notation is used in this paper:

T	Temperature, $°C$.
$T_{(i)}$	Term of i-th degree in a power expansion of T.
t	Time, sec or $10^6 yr$.

p	Density, $g\ cm^{-3}$.
c	Specific heat, $cal\ g^{-1}(°C)^{-1}$.
k	Thermal conductivity, $cal\ cm^{-1}sec^{-1}(°C)^{-1}$.
α	Thermal diffusivity, cm^2sec^{-1}.
Q	Rate of heat flow per unit area, $cal\ cm^{-2}sec^{-1}$.
m	Fraction of the total heat generated in the crust which is conducted to the earth's interior.
q	Rate of radiogenic-heat generation, $cal\ cm^{-3}sec^{-1}$.
q_1	Rate of radiogenic-heat generation for first layer, $cal\ cm^{-3}sec^{-1}$.
q_2	Rate of radiogenic-heat generation for second layer, $cal\ cm^{-3}sec^{-1}$.
z	Depth, measured from earth's surface, positive downward, cm or km.
z'_0	Initial depth of a given horizon of the substratum, measured from the base of the crust, positive downward, cm or km.
z'	Final depth of a given horizon of the substratum, measured from the base of the crust, positive downward, cm or km.
a	Depth-attenuation coefficient of the subsidence, cm^{-1}.
$\boldsymbol{h_0}$	Initial depth of the base of the first layer, cm or km.
h	Final depth of the base of the first layer, cm or km.
$\boldsymbol{H_0}$	Initial depth of the base of the second layer, cm or km.
H	Final depth of the base of the second layer, cm or km.
S	Subsidence, cm or km.
D	Smallest of the distances from a point to the two faces of a plane parallel slab, cm or km.
D	Largest of the distances from a point to the two faces of a plane parallel slab, cm or km.
Z_1	Reduced value of ; $z_1 = z/2(\alpha t)^{1/2}$,dimensionless. The addition of the sub-index 1 to a length symbol will denote division by $2(\alpha t)^{1/2}$.
u	Reduced variable, $u = \zeta/2(\alpha t)^{1/2}$.
E	Steady-state temperature gradient when the heat comes from the deep interior, $°C\ cm^{-1}$ or $°C\ km^{-1}$.
T_1	Ultimate temperature increase because of subsidence for $0 \le z \le s$,when the heat comes from the deep interior, $T_1 = Es$.
ΔT^I	or $\Delta \boldsymbol{T^{II}}(z,t)$**.** Temperature function, which at $t = 0$ is equal to $-T_1$ in the interval $0 \le z \le \infty$; and which is $\Delta T^I = 0$ for $z = 0$ and $t > 0$.
ΔT^{II}	or $\Delta T^{II}(z,t)$. Temperature function, which at $t = 0$ is a linear function of z between the point $z = 0$, $\Delta T^{II} = T_1$ and the point $z = s$, $\Delta T^{II} = 0$; and which is $\Delta \boldsymbol{T^{II}} = 0$ for $z = 0$ and $t > 0$.
ΔT^{III}	or $\Delta T^{III}(s,t)$. Temperature function, which at $t = 0$ is equal to the deviation from $T = Ez$ produced by the distortion of the substratum.
ΔT_a	or $\Delta T_a(z,t)$**.** Temperature deviation from the final steady state, when the heat comes the deep interior.
ΔT_A	or $\Delta T_A(z,t)$**.** Temperature variation from the initial steady state, when the heat comes from the deep interior.
T_2	Ultimate temperature increase in the substratum, when the heat is generated in the crust
ΔT_b^I	or $\Delta T_b^I(\boldsymbol{z},\boldsymbol{t})$**.**Temperature function equal at $t = 0$ to the initial deviation from the initial steady state, when the heat is generated in the crust.

ΔT_b^{II} or $\Delta T_b^{II}(\boldsymbol{z},\boldsymbol{t})$. Temperature build-up produced by the radiogenic heat sources added by the subsidence.

ΔT^{IV} or $\Delta T^{IV}(\boldsymbol{z},\boldsymbol{t})$. Temperature function, which at $t = 0$ is a linear function of z between the point $z = 0$, $\Delta T^{IV} = \boldsymbol{T_2}$ and the point $z = s$, $\Delta T^{IV} = 0$ and which is $\Delta T^{IV} = 0$ for $z = 0$ and $t > 0$.

ΔT^{v} or $\Delta T^{V}(z,t)$**.** Temperature function, which for $t = 0$ is a linear function of z between the point z = 0, $\Delta T^{V} = T_2$ and the point $z = H_0 + s/2$, $\Delta T^{V} = 0$; and which is $\Delta T^{V} = 0$ for $z = 0$ and $t > 0$.

ΔT_b or $\Delta T_b(z,t)$**.** Temperature deviation from the final steady state, when the heat is generated in the crust.

$\boldsymbol{\Delta T_B}$ or $\boldsymbol{\Delta T_B}(z,t)$ $(\boldsymbol{z},\boldsymbol{t})$ Temperature variation from the initial steady state, when the heat is generated in the crust.

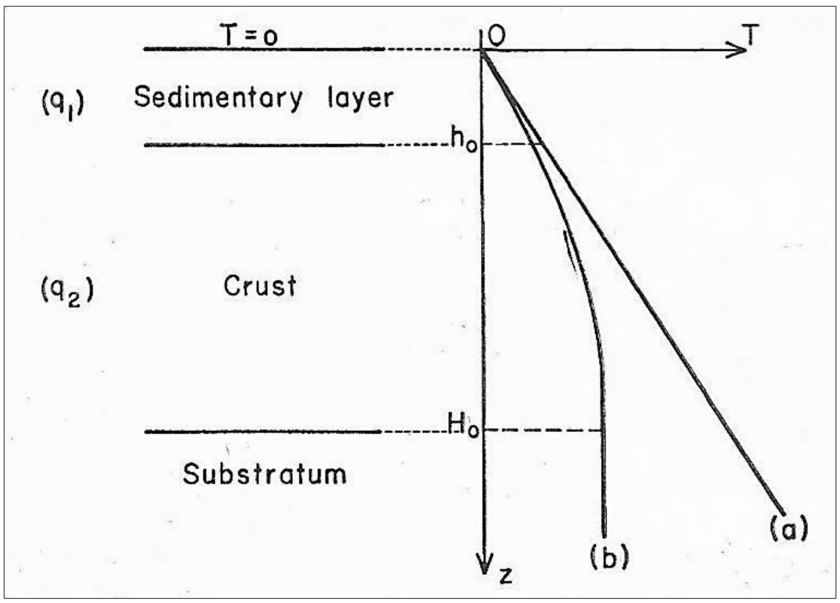

Figure 1. Types of Steady Temperature Distribution, (a) due to Heat from the Deep Interior, (b) due to Heat Generated in the Crust

Statement of problem

If the conditions in the vertical plane through the middle of a basin are considered, determination of the temperature variations owing to sedimentation and subsidence can be regarded as a one-dimensional problem. This is because the width of a basin is several times the crustal thickness, and its length is again large in comparison to its width.

This paper, therefore, considers three horizontal layers - the top one representing the sedimentary layers, the middle one representing the crystalline part of the continental crust, and the lowest one a plastic substratum which can be assumed to extend downward to infinity. The thermal conductivity and diffusivity are taken to be the same for the three layers. It is assumed that the top surface of the uppermost layer is kept at a constant temperature (which may be taken as zero) and that before the subsidence, the temperature distribution has reached the steady state. The three layers are shown in Figure 1, as well as two alternatives, (a) and (b), for the initial temperature distribution. The upward flow of heat from the deep interior, shown at (a), may represent either a gradual cooling of an originally hot earth or may be due to other deep heat sources. It will be referred to as heat from the deep interior. For alternative (b) the radioactive sources of heat are assumed to be distributed with different but uniform densities in the two uppermost layers.

At a particular instant the thickness of the top layer may increase, owing to deposition of sediments, assumed to be laid at zero temperature- If the topmost surface is taken as a level of reference, then the two topmost layers are displaced downward. If the subsidence occurs sufficiently quickly, the initial temperature distribution will be bodily displaced with the material but otherwise will be practically unchanged. This will be termed rapid subsidence. If the subsidence occurs slowly, the temperature distribution will readjust continuously.

The equations satisfied by the temperature must be considered. For one-dimensional heat flow in an isotropic rigid body with internal heat sources, the temperature satisfies the equation given by Carslaw and Jaeger (1948, p. 59, equation 1)

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} = \alpha \frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{t}} + \frac{q}{\rho c}, \tag{1}$$

where a equals $k/\rho c$ equals thermal diffusivity, k equals thermal conductivity, ρ equals density, c equals specific heat, and q equals rate of heat generation per unit volume.

When the body changes shape simultaneously with heat flow, equation 1 must be modified. If, around a volume V, a closed surface S is displaced by the distortion of the body, the balance of heat across S gives the following equation:

$$\int_{(s)} k \; grad \; T \, . dS = \frac{d}{dx}\left\{\int_{(V)} \rho c \; T \; dV\right\}. \tag{2}$$

In this equation d/dt represents differentiation following the elements of the volume. When the volume V is very small, the equation reduces to

$$k \; div \; grad \; T = \rho c \left(\frac{\partial T}{\partial t} + \frac{\partial T}{\partial s}\frac{ds}{dt}\right), \tag{3}$$

where ds is the displacement in the direction of the motion. The density p is taken to be independent of time. From equation 3,

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{x}} + \frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{y}} + \frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{z}}\right) - v\frac{\partial T}{\partial t}, \tag{4}$$

where v is the velocity, When the problem is two-dimensional, both with respect to heat flow and distortion, one of the second derivatives of T can be made zero by choosing one axis, say y, along the length. Furthermore, in the particular case of a subsiding basin, another axis, say z, can be chosen in the vertical plane along its axis. On the axial zone, the isotherms will be very nearly horizontal planes, hence, $\frac{\partial^2 \mathrm{T}}{\partial x^2} \simeq 0$. Also, the velocity will have only z components. Therefore, according to Carslaw and Jaeger (1948, p. 127).

$$\frac{\partial T}{\partial t} = \frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{z}} + v\frac{\partial T}{\partial z}. \tag{5}$$

Theory of rapid subsidence

General considerations

The model in which the heat comes from the deep interior will be considered first. For cooling of a once-hot earth, it can be verified that after $2000\ million\ years$ the gradient is practically constant down to a depth of more than $200\ kilometers$.Only that part of the temperature profile above about $150\ kilometers$ depth is really important for the determination of the temperature variations. Hence, there is no serious error in assuming a uniform gradient, denoted here by $E,$ to represent the heat flow coming from the deep interior.

The subsidence of the crust is accompanied by a downward displacement of the initially linear temperature distribution. Since the deformation and flow of the substratum must tend to zero with increasing depth, the temperature at great depth is not affected initially by the subsidence. Therefore, the initial temperature distribution $T = Ez$, is replaced by a distorted distribution, which corresponds to the line OAB and the curve BC in Figure 2A, (1). The curve BC at sufficiently great depth approaches the original distribution.

The distorted distribution can be replaced by (1) distribution $= Ez$, plus (2) deviation ΔT_a . The first distribution represents a steady state which is maintained by the new heat that comes from the hot interior and does not need further study. The second is a transient effect that gradually decays to zero. The transient effect will be discussed hereafter.

In the other model, the heat is produced by radioactivity of the crust. For the analysis of the problem a distinction should be made between old and new radiogenic heat. The old heat is in the ground prior to the subsidence; the new heat is generated after the subsidence. The old radiogenic heat is represented by the initial temperature distribution. The sedimentary layer is assumed to have a homogeneous distribution of radiogenic heat sources $\boldsymbol{q_1}$, and the crust a similar kind of distribution, $\boldsymbol{q_2}$ (Figure 1). The error introduced by assuming that $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ are $constants$ is very small because the half lives of the radioactive elements involved are much larger than the times of interest here.

The steady-temperature distribution produced by the radioactive sources is obtained by solving the differential equation

$$k\ \frac{\mathrm{d}^2\mathrm{T}}{\mathrm{d}^2\mathrm{z}} + \mathrm{q} = 0. \tag{6}$$

There are five boundary conditions to satisfy, namely $T = 0$ at $z = 0$, $[T] = 0$ at $z = h_0$ and at $z = H_0$ and $\left[k\ \frac{\mathrm{d\,T}}{\mathrm{d\,z}}\right] = 0$ at $z = h_0$ and at $z = H_0$.The symbol [ ] denotes the change across a boundary of the quantity enclosed by the symbol. The integration of equation 6 introduces six arbitrary constants. Since the boundary conditions are only five, one arbitrary constant remains undetermined. This constant corresponds to flow of heat coming from below the radioactive layers. The part of the solution which corresponds to the radiogenic heat only is

$$T = \frac{q_1 Z^2}{2k} = \frac{(h_0 q_1 + H_0 q_2 - h_0 q_2)z}{k}, \tag{7a}$$

$$T\ =\ -\frac{q_2\, Z^2}{2k} + \frac{q_2 H_0 z}{k} + \frac{h_0^2(q_1 - q_2)}{2k}, \tag{7b}$$

$$T = \frac{(h_0^2 + H_0^2 q_2 - h_0^2 q_2)}{2k}, \tag{7c}$$

for the first, second, and third layers, respectively. These equations describe the initial equilibrium-temperature distribution which is rigidly displaced by the subsidence. Since no radioactive sources are assumed to exist in the substratum, the equilibrium temperature there is constant. Hence, there is no distortion of the temperature profile due to the deformation of the substratum.

The displaced temperature distribution can be considered as the superposition of the initial plus a transient distribution $-\Delta T_b^I$ (Figure 2 B). Furthermore,the subsidence alters the distribution of radioactive matter because of the displacement of the crust and of the thickening of the sedimentary layer. This alteration is
equivalent to the addition of two radioactive slabs, one extending from $z = h_0$ to $z = h_0 + s$ and with sources $(q_1 - q_2)$ and the other extending from $z = H_0$ to $z = H_0 + s$ and with sources q_2 as shown in Figure 2B.

Therefore, the total deviation $\Delta \boldsymbol{T_b}$from the new equilibrium condition is the sum of (1) transient term $\boldsymbol{\Delta T_b^I}$ and (2) what is left of the ultimate temperature build-up, namely $-|\Delta T_b^{II}|$, produced by the new heat sources. The symbol ΔT_b^{II} denotes the temperature build-up produced by these sources.

Model in which the heat comes from the deep interior

Initial conditions - In the model in which the heat comes from the deep interior, the temperature variation, OABC at (1) in Figure 2A, immediately after a subsidence s, is the superposition of the steady-temperature distribution $T = Ez$, plus the temperature distributions $\Delta T^I(z,0)$, $\Delta T_b^{II}(z,0)$, and $\Delta T_b^{III}(z,0)$ as shown at (2) and (3) in Figure 2A. The symbol $\Delta T^{II}(z,t)$ denotes the temperature function which is the decay of an initial distribution $\Delta T^I(\text{z},0)$. The sum $(\Delta T^I + \Delta T^{II})$ represents the perturbation produced by the surface, and ΔT_b^{III} represents the perturbation produced by the base of the crust and by the deformation of the substratum.

The definitions of ΔT^I, ΔT_b^{II} and ΔT^{III} for $t = 0$ are as follows: (1) $\Delta T^I(z,0)$, is a function which has a constant value $-T_1$ from $z = 0$ to $z = \infty$; (2) $\Delta T^{II}(z,0)$ is a linear function of z which extends from $z = 0$, where it has the value T_1, to $z = s$, where it is zero, and which is zero elsewhere; that is,

$$\Delta T^{II}(\text{z},0) = T_1\left(1 - \frac{z}{s}\right),$$

$$(0 \leq \mathbf{z} \leq s);$$

and (3) $\Delta T^{III}(z,0)$ is a function which results from the deformation of the substratum; it extends over the range $z \geq H$.

Determination of $\Delta \mathbf{T^I}$ - The term ΔT^I is easy to consider. The temperature distribution in a semi-infinite solid whose surface is kept at zero temperature and whose initial temperature is $-T_1$(constant) is given by Carslaw and Jaeger (1948, p. 41, equation 4) as

$$\Delta T^I = -\boldsymbol{T_1\, erf\;\, z_1}, \tag{8}$$

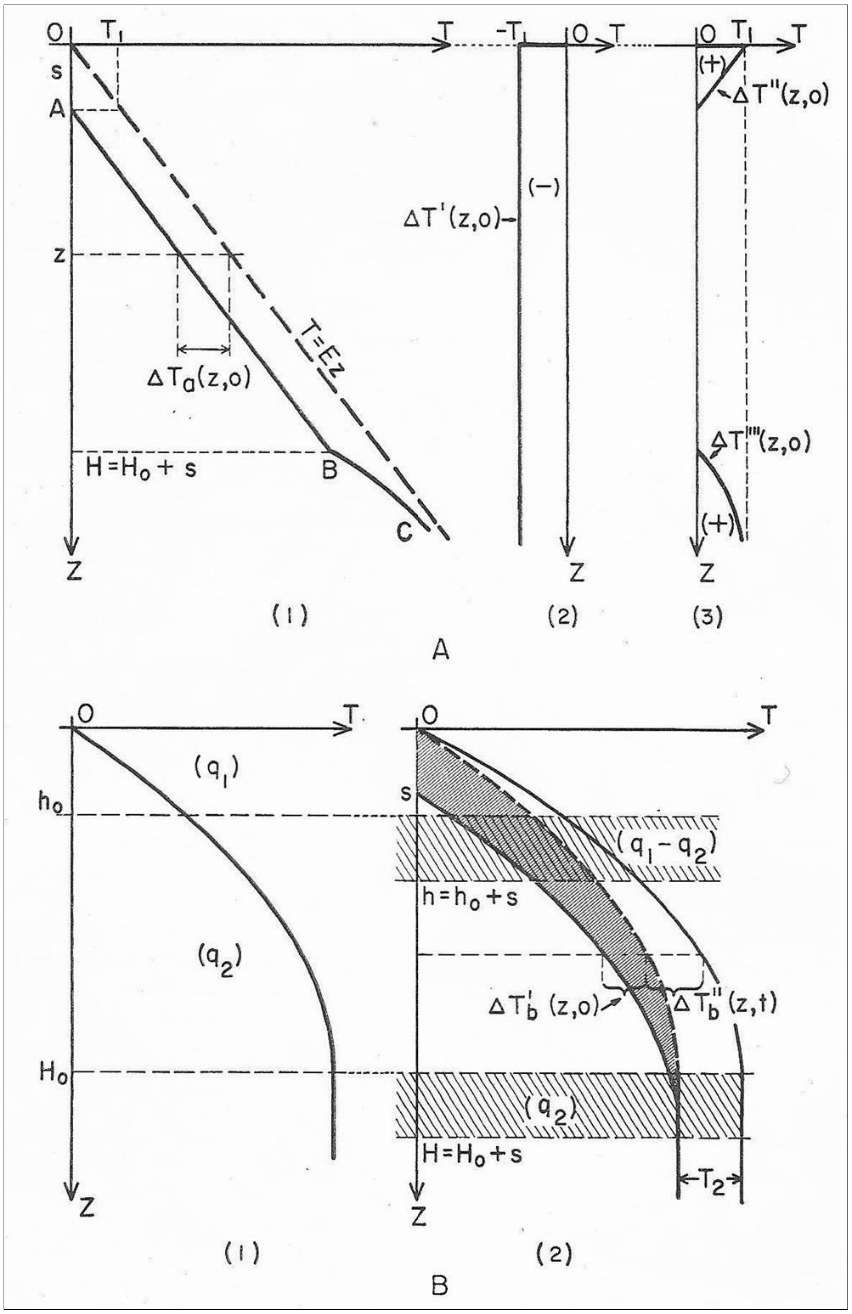

Figure 2 . Effect of a Rapid Subsidence on Temperature Distribution**.** In A, heat comes from the deep interior; the deviation from the initial condition is resolved in partial terms shown at **(2)** and **(3).** In B, neat comes from a radiogenic source; **(1)** shows initial condition, (2) shows displaced-temperature distribution.

where $z_1 = z/2(\alpha t)^{1/2}$.Throughout this paper, a sub index 1 is added to a length symbol to denote division by $2(\alpha t)^{1/2}$.The term T_1 is $T_1 = Es$.

Determination of ΔT^{II} - The value of ΔT^{II} - is determined by considering an infinite solid with an initial temperature distribution $f(z)$ equal to $T_1(1 - z/x)$ in the interval $0 \le z \le s$ and equal to $-T_1(1-z/x)$ in the interval $0 \geq z \geq -s$. With these two distributions, the temperature at $z = 0$ is constantly zero, as required by one of the boundary conditions. The temperature in an infinite solid, when the initial temperature distribution is $f(z)$, is given by Cars law and Jaeger (1948, p. 34, equation 1) as

$$T = \frac{1}{2\sqrt{\pi\alpha t}} \int_{-\infty}^{+\infty} f(z^1)\ .exp\left\{\frac{-(z-z^1)^2}{4\alpha t}\right\} dt \tag{9}$$

If $\boldsymbol{f(z)}$ is replaced by the function indicated and the equation is integrated,

$$\Delta T^{II} = T_1 \, {\{i^1 erfc(z-s)_1 - i^1 erfc(z+s)_1\}}\Big/{2s_1} - T\ erfcz_1, \tag{10}$$

where $\boldsymbol{z^1 = z/2(\alpha t)^{\frac{1}{2}}}$, etc., and $\boldsymbol{i^1}$ erfc is the first integral of the error function (Hartree, 1935, p.85).

APPROXIMATION FOR LARGE t -An initial temperature distribution $\boldsymbol{f(z)}$ and the corresponding anti symmetrical one in order to have $\boldsymbol{T = 0}$ at $\boldsymbol{z = 0}$ are now considered. If the distribution is anti symmetrical and it extends only throughout finite values of $\boldsymbol{z}$, then the asymptotic value of the temperature obtained from equation 9 is

$$T = \frac{zM}{2\sqrt{\pi}(\alpha t)^{3/2}}, \tag{11}$$

Here M is the first moment with respect to the $z = 0$ plane of that part of the temperature distribution $f(z)$ lying on the positive side of $\boldsymbol{z = 0}$. Hence, for large, the distribution becomes linear in $\boldsymbol{z}$ and decays as $\boldsymbol{i^{-3/2}}$. For the particular case of ΔT

$$\Delta T^{II} = 2T_1 x_1^2 z_1 / 3\sqrt{\pi}. \tag{12}$$

Determination of ΔT^{III} - Determination of the actual deformation of the substratum produced by the subsidence of the crust requires consideration of the shape assumed by the crust and also the variation, in the substratum, of the viscosity and the strength with depth. Furthermore, the melting point may be reached at a depth of about **80 *kilometer*** (Gutenberg, **1951**).

These complexities can be avoided if it is assumed that the displacement at a certain depth, corresponding to a small subsidence of the crust, decreases exponentially with the depth below the crust and further that if the crust is progressively displaced downward, the small incremental displacements are described by the same law. This kind of deformation requires thinning in the $\boldsymbol{z}$ direction and spreading in the horizontal directions.

If the vertical displacements are referred to a vertical axis $\boldsymbol{z'}$ directed downward, with its origin at the base of the crust, and Δs is an elemental subsidence of the crust, then

$$\Delta z' = -\Delta s\{1 - exp(-az')\}. \tag{13}$$

The parameter a introduced in this equation is a depth-attenuation coefficient. The integration of $\boldsymbol{\Delta s}$, as given by equation 13, from $S = 0$ to $S = s$, after the result is transformed, gives

$$z_0 = z + \frac{1}{a}\log[1 + \{\exp(-as) - 1\}.\exp\{-a(z-H)\}], \tag{14}$$

where now z_0 and z are the initial and final depths, respectively, below the earth's surface of a layer P (Figure 3). For a rapid subsidence, a temperature T at depth so is carried to depth z.That is, if $\boldsymbol{T = Ez_0}$is the initial temperature distribution, then after the subsidence

$$\boldsymbol{T = Ez + \frac{E}{a} log[1 + \{exp(-as)\} - 1 . exp\{-a(z-H)\}]}. \quad (15)$$

The second term on the right side is the distortion with respect to the initial steady state. The deviation $\boldsymbol{\Delta T^{III}}$ with respect to the final steady state is Es minus this distortion, i.e.,

$$\boldsymbol{\Delta T^{III}} = \frac{\mathrm{E}}{\mathrm{a}} \log[\exp \mathrm{as} - (\exp \mathrm{as} - 1) . \{-\mathrm{a}(\mathrm{z} - \mathrm{H})\}\,] . \quad (16)$$

This term is positive, as shown at (3) in Figure 2A.

At $\boldsymbol{t} = 0$, the distribution ΔT^{III} for $z \geq$ H is considered, and, so that the boundary condition $\Delta T^{III} = 0$ at $\boldsymbol{z} = 0$ may be satisfied, its anti symmetrical for $\boldsymbol{z} \leq -H$ is added. With these two distributions, the temperature at any time t can be deduced from equation 9. The argument of the logarithm in equation 16 is denoted by $(1 + \boldsymbol{x})$.It is easy to verify, for the range of values of the different quantities which enter into it, that x is at most a fraction of 1. Thus, the approximation logs $(1 + \boldsymbol{x}) \simeq \boldsymbol{x}$ can be used. With this approximation the integration of equation 9 gives

$$\Delta T^{III} = T_1\{\theta(H + z, t) - \theta(H - z, t)\}, \quad (17)$$

where

$$\theta(z, t) = {}^1\!/_2 \left[\exp(as + a^2\alpha t) .\ \{1 - \mathrm{erf}(a\sqrt{\alpha t}) + \mathrm{erf}\, {}^Z\!/_2 \sqrt{\alpha t}\}\right]. \quad (18)$$

A useful asymptotic value of $\boldsymbol{\Delta T^{III}}$, obtained by expanding $\boldsymbol{\theta}(z, t)$ in powers of $(\alpha t)^{-1/2}$and letting t increase, is

$$\boldsymbol{\Delta T^{III} = T_1}. \left[\frac{z}{\sqrt{\pi\alpha t}} - \frac{az^3 + 3H(2 + aH)z}{12a\sqrt{\pi}(\alpha t)^{3/2}}\right], \quad (19)$$

Furthermore, the asymptotic value of the surface gradient is

$$\left(\frac{d}{dz}\Delta T^{III}\right)_{z=0} = T_1. \left[\frac{1}{\sqrt{\pi\alpha t}} - \frac{H(2 + aH)}{4a\sqrt{\pi}(\alpha t)^{3/2}}\right]. \quad (20)$$

Therefore, for large t, ΔT^{III} varies as $t^{-1/2}$, i.e., less rapidly than ΔT^{III}, which varies as $\boldsymbol{t^{-3/2}}$.

Model in which the heat is generated in the crust

Transient temprature distribution ΔT_b^I - In the analysis of the effect of rapid subsidence on the model in which heat is generated in the crust, it was found that a part of the disturbance is a transient temperature distribution ΔT_b^I (Figure 2B).

Prior to the subsidence, the temperature distribution is assumed to have reached the steady state; this is described by equations 7a-7c. The temperature distribution in the crust, when the initial distribution is rigidly displaced through a distance *s,* is obtained by replacing z by

$(Z - S)$ in equation 7b. Therefore, in the range $s \leq z\, H_0$, the difference ΔT_b^I of the two distributions is a linear function of z, namely

$$\Delta T_b^I = -\frac{q_2 s}{k} z + \frac{q_2 s}{2k}(s + 2H_0), \quad (21)$$

and is represented by the line AB in Figure 4. It is also practically linear in the range $0 \leq z \leq s$.

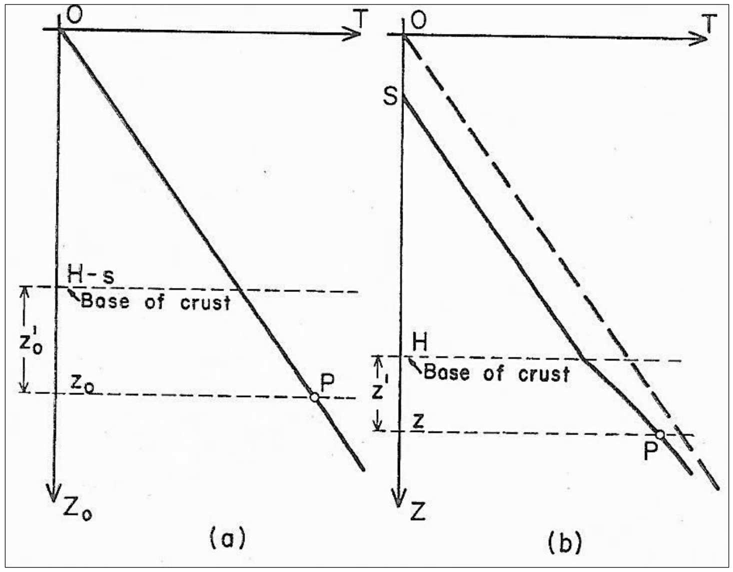

Figure 3 . Distortion of the Temperature Profile in the Substratum after ***A*** Rapid Subsidence

The distribution described by equation 21 gives $\Delta T_b^I = 0$ at $z = H_0 + s/2$. In the interval $Ho \leq z \leq H$, ΔT_b^I is an arc of parabola, arc BC, which is tangent to the $z - axis$ at C. The arc BC differs very little from the broken line BDC.

Thus, ΔT_b^I at $t = 0$ can be considered as the difference of the triangular distributions $\Delta T^V(z, 0) = \overline{ODF}$ and $\Delta T^{IV}(z, 0) = \overline{OAF}$. Since the initial form of ΔT^{II} is also triangular, the time decay of the two distributions can be obtained from equation 10 by replacing T_1 with T_2 and by keeping *s* for and replacing s by $(H_0 + s/2)$ for ΔT^V,

The term T_2, which is equal to $\overline{OF}$ in Figure 4, is obtained by making $z = 0$ in equation 21, which gives

$$T_2 = \frac{q_2 s(s + 2H_0)}{2k}. \quad (22)$$

The term T_2 is equal to the ultimate temperature increase in the substratum if $q_1 = q_2$. Also, it has another interpretation. It is easily shown that the steady-state temperature below a slab with heat-source intensity q_2 extending from $z = H_0$ to $z = H_0 + s$ is equal to T_2.

In the other model the temperature variations were expressed in terms of $T_1 = Es$ instead of T_2. If the two models are required to give the same thermal gradient at the surface, the following relationship exists:

$$T1 \;=\; \frac{H_2-(1-{}^{q_1}/_{q_2})h_0}{H_0+{}^{s}/_{2}}T2. \tag{23}$$

Since $s/2 \leq H_0$ and $q_1/q_2 \simeq T_2$ is not too different from T_2 . For this reason, the temperature variations for the two models can be compared directly when expressed in terms of $T_1$and T_2.

Radioactive slads - In the preliminary analysis of the radiogenic model the effect of the alteration of the initial distribution of radioactive matter was mentioned. This alteration consists in the addition of two slab distributions of radioactivity, as shown in Figure 5.

The temperature distribution generated by these slabs can be determined by considering a horizontal slab of uniform thickness which extends from z $= h$ to $z = H$, which contains heat sources of intensity q and which is in a semi-infinite medium. The following boundary and initial conditions exist: $T = 0$, $z = 0$ for all z. The effect of the slab can be obtained from the solution for a plane source of heat of infinitesimal thickness, placed at depth z', and which starts to generate heat at $t = 0$, (Carslaw and Jaeger, 1948, p. 222, equation 8), namely

$$T = q\left(\frac{t}{\pi\alpha}\right)^{\frac{1}{2}} exp\{-(z-z')^2_1\} - \frac{q|z-z\prime|}{2\alpha} erfc\;|z-z'|_1. \tag{24}$$

In the notation of this formula (pcq) is the rate of heat generation per unit area of the plane source, which is equivalent in the notation of this paper to qdz' (dz' *is the thickness of the elementary slab*). Formula 24 is valid for an infinite body and satisfies the initial condition $T = 0$ at $t = 0$.

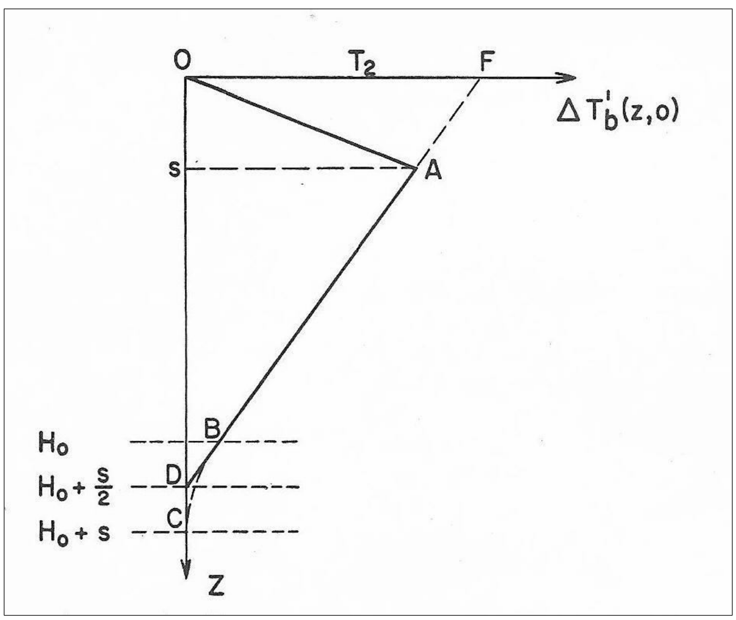

Figure 4 . Transient Distribution ΔT_b^I at $t = 0$.

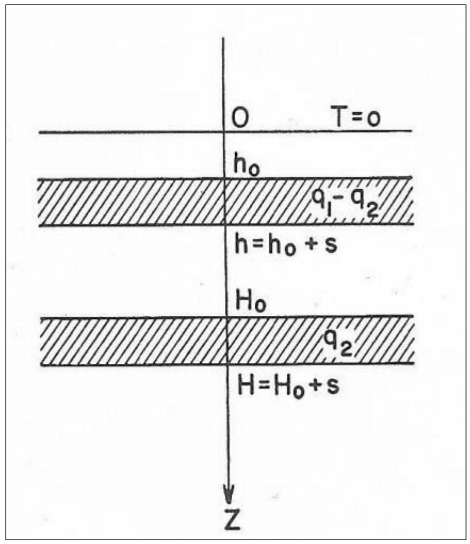

Figure 5 . Radioactive Slabs

The temperature produced by a slab extending from z $= h$ to $z = H$ in an infinite body is obtained by integration of the temperature produced by the elementary slab. The expression $|z — z'|$ erfc $|z — z_1'|$ IS in all cases positive and is a function of the distance of the point where the temperature is measured to the elementary slab. Then, the effect of the term that contains it is symmetrical with respect to the slab. To avoid confusion of signs it is better to express the results in terms of the greatest and the smallest distances D and d, respectively, of the plane P where the temperature is measured to the faces of the slab. If P is inside the slab, D is the distance to the face chosen as upper, as shown in Figure 6.

When P is outside the slab, either Pi or Pg in Figure 6, then

$$T = \frac{2qt}{\rho c}\left(i^2 erfc\, d_1 - i^2 erfc\, D_1\right), \qquad (25)$$

when P is inside the slab,

$$T = \frac{qt}{\rho c}(1 - 2i^2 erfc\, D_1 - 2i^2 erfc\, d_1), \qquad (26)$$

where i^2 $erfc$ $is\ the\ second\ repeated\ integral\ of\ the\ error\ function$, and $D_1 = D/2(\alpha t)^{1/2}$, etc.

To satisfy the boundary condition at $z = 0$, a slab is added which is the image with respect to the $z = 0$ plane of the $(+q)$ slab, but of intensity $(— q)$. Therefore,

$$T = \frac{2qt}{\rho c}\{i^2 erfc\, d_1 - i^2 erfc\, D_1 + i^2 erfc(h + H + d)_1 - i^2 erfc(h + H - D)_1\}, \qquad (27)$$

for the temperature between the surface and the slab (point P_1, Figure 6);

$$T = \frac{qt}{\rho c}\{1 - 2i^2 erfc\, D_1 - 2i^2 erfc\, d_1 + 2i^2 erfc(h + H + D)_1 - 2i^2 erfc\,(h + H - D)_1\}, \qquad (28)$$

for the temperature inside the slab (point P_2, Figure 6); and

$$T = \frac{2qt}{\rho c}\{i^2 erfc\, d_1 - i^2 erfc\, D_1 + i^2 erfc(h+H+D)_1 - i^2 erfc\,(h+H-D)_1\}, \tag{29}$$

for the temperature below the slab (point P_3, Figure 6)

For large t, *i.e.,* small values of μ, it is useful to expand i^2 *erfc* μ in the power series

$$i^2 erfc\, u = \frac{1}{4} - \frac{u}{\sqrt{\pi}} + \frac{u^2}{2} - \frac{u^3}{3\sqrt{\pi}} \ldots \tag{30}$$

In equations 25-29, the $i^2 erfc$ terms are multiplied by $2qt/pc$. Hence,when $t \to \infty$,the terms in the first power of $''u''$ increase without bounds. Since these terms do not cancel out in equations 25 and 26, the temperature increases indefinitely with time when the medium is infinite. However, the u terms in equations 27, 28, and 29 cancel out. Therefore, for a semi-infinite medium, with the boundary condition $T = 0$ at $z = 0$, the temperature does not increase indefinitely.

The u^2 terms are independent of time, and they must represent the steady state. In fact, it is easy to verify that the u^2 terms in equations 27, 28, and 29 lead to equations $7a - 7c$, after allowance for changes of notation. The u^3 terms decrease as $t^{-\frac{1}{2}}$ on separating these terms, the same value

$$T_{(3)} = -\frac{q}{k\sqrt{\pi}}(H^2 - h^2)z_1, \tag{31}$$

is found for the temperatures above, inside, and below the slab. The subscript (3) denotes the part of T corresponding to the terms. For a given time, $T_{(3)}$ varies linearly with z .

Since the coefficient of u^4in the series of equation 30 is zero, the addition of $T_{(3)}$to the steady state gives quite accurate values of the temperature, even for relatively large u. This is well illustrated in Figure 11, where the deviation from the steady state for $t = 50 \times 10^5\ yr$ and greater times is practically linear in *z*. The formulae derived for the radioactive slab are of course applicable to heat sources other than radioactive heat sources.

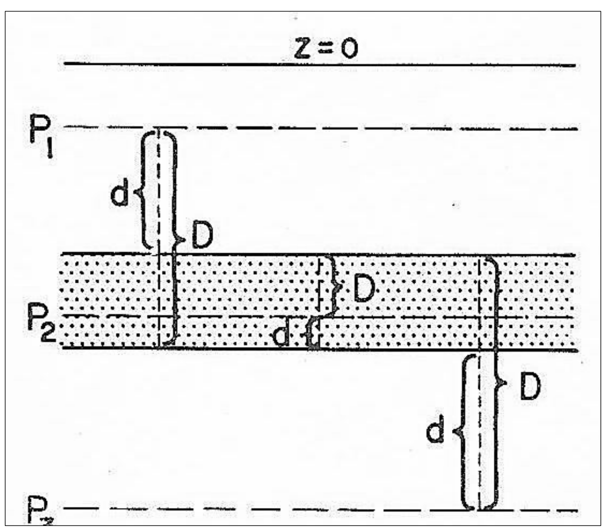

Figure 6 . Convention about Distances D and d

Radiogenic heat conducted to the earth's interior - Inspection of the temperature build-up curves for radioactive slabs close to the earth's surface (Figure 10, 11) demonstrates that most of the heat generated during the life of the earth must have been lost at the surface.

A layer with sources of intensity q, which extends from the surface to a depth H, may be assumed to represent the radioactive part of the crust. The heat conducted to the surface per unit of horizontal area is

$$Q(t) = \int_0^t k(\partial T/\partial z)_{z=0}\, dt. \qquad (32)$$

By introducing $\left(\frac{\partial T}{\partial z}\right)_{z=0}$ derived from equation 27, carrying out the integration, and expanding in powers of $H_1 = H/2\,(\alpha t)^{1/2}$, one can find for long times that the heat conducted to the interior, expressed as a fraction of the total heat, is

$$m = \frac{2}{\sqrt{\pi}} H_1, \qquad (33)$$

Table 1 – values of principal parameters.

Parameter	Case 1	Case 2
Subsidence, s	6×10^5 **cm**	$13 \times 10^5 cm$
Initial thickness of sediments, h_0	$2 \times 10^6 cm$	$2 \times 10^6 cm$
Initial depth of base of crust, H_0	$32 \times 10^5 cm$	$32 \times 10^5 cm$
Thermal diffusivity, α	$0.010\ cm^2 sec^{-1}$	$0.010\ cm^2 sec^{-1}$
Thermal conductivity, k	$0.006\ cgs$	$0.006\ cgs$
Depth-attenuation coefficient of the subsidence,a	$0.23 \times 10^{-6}\ cm^{-1}$	$0.23 \times 10^{-6}\ cm^{-1}$

If the age of the crust is $2000 \times 10^6\ yr$ and $\alpha = 0.010\ cm^2 sec^{-1}$ and $H = 30 \times 10^5\ cm$, then $m = 0.067$. Hence, only about 7 per cent of the total heat generated in the crust has been conducted to the interior. It can readily be shown that this heat corresponds to an average temperature rise of the whole earth equal to

$$\Delta T = \frac{3qHim}{\rho c R}. \qquad (34)$$

The new symbols introduced here are R, the radius of the earth, andΔT, the average rise in temperature. Formula 34 assumes that the radioactive crust extends all over the surface of the earth. If the values of H and t are introduced, and also $m = 0.067$, $\rho = 5.52\ g\ cm^{-3}$, $c = 0.20$ cal g^{-1}°C^{-1}, and $R = 6.37 \times 10^8 cm$, then $\Delta T = 22$°C. The average increase of temperature throughout the earth is only one-fifteenth of the maximum increase (which is attained at' the base of the crust).

Therefore, the heating produced by the radioactive crust is practically negligible at the central zone of the earth even after $2000\ million\ years$. This indicates that, as far as the heat produced in the crust is concerned, the error arising from the assumption that the earth's surface is a plane and that the earth extends to infinity in one direction is not serious.

Application or rapid-subsidence theory to a basin

General considerations

As an application of the rapid-subsidence theory, the temperature variations for specific subsidences of 6 and 13 km are calculated on the assumption that initially there is a layer of 2 km of sediments resting on a crystalline crust 30 km thick (Figure 1). The same value, 0.010 $cm^2 sec^{-1}$, is taken for the thermal diffusivities of the three layers. The assumed values of the different parameters are summarized in Table 1. The temperature variations are calculated at times of 0.1,1,5,20,50,100and 300 $million\ years$ after the subsidence. For comparison of the results for the two models, both are assumed to produce the same geothermal gradient at the surface.

Components of temperature deviation when the heat comes from the deep interior

It has been shown that when the heat comes from the deep interior the subsidence produces a temperature deviation from the final steady state which is

$$\Delta T_a = \Delta T^I + \Delta T^{II} + \Delta T^{III}. \quad (35)$$

The term $\Delta T^I(z,t)$ at $t = 0$ has the constant value $-T_1$from $z = 0$ to $z = \infty$ .The term $\Delta T^{III}(z,t)$ at $t = 0$ is linear in z and extends from $z = 0$ where it has a value T_1 to $z = s$ where it is 0. Finally, the term $\Delta T^{III}(z,t)$ is produced by the deformation of the substratum. These temperature variations have been expressed in terms of $T_1 = Es\ as\ unit,\ \ where\ E\ is\ the\ steady - state\ gradient\ and\ s\ is\ the\ subsidence$. The temperature T_1 is the ultimate temperature increase throughout the crust. The variation ΔT_a , written in full,

$$\Delta Ta = -T_1 \{i^1\text{erfc}(z - s)_1 - i^1\text{erfc}(z + s)^1\}.T_1/2s_1 + \{\theta(h+z,t) - \theta(h - z.t)\}T_1, \quad (36)$$

where i^1erfc is the first integral of the error function, θ is a function defined under Determination of ΔT^{III}, $z^1 = z/2(\alpha t)^{1/2}$, and $s^1 = s/2(\alpha t)^{1/2}$. The depth z is measured from the surface; H is the depth of the base of the crust.

For long times, the asymptotic value of ΔT_a is

$$\Delta T_a = -\frac{Es(6H + 3aH^2 - as^2)}{12a\sqrt{\pi}(\alpha t)^{3/2}}. \quad (37)$$

Thus, after a long time, the temperature variation behaves as $t^{-3/2}$. The factor a is an attenuation coefficient of the subsidence; a is taken as $0.23 \times 10^{-6} cm^{-1}$, which corresponds to an attenuation of the subsidence at 100 $kilometer$ below the crust to one-tenth of its value. When t and a are large, the temperature variation becomes independent of a. When a is small, the variation increases as $1/a$.

The propagation of temperature disturbances in the earth is clarified by examination of the components of ΔT_a. For this discussion,T_1 is taken as 1.

The disturbance produced by the surface is presented by $(\Delta T^I + \Delta T^{II})$. Initially, this term increases linearly from 0 at the surface to 1 at a depth equal to the subsidence. Below this depth, it is equal to 1. The decay of this surface perturbation is illustrated in Figure 7 for subsidences of 6 and 13 km. The initial break in the gradient is smoothed out very quickly. In fact, in only

$100{,}000\ years$, it is smoothed throughout a depth range of about 10 kilometer. After about 5 million years, the difference between the curves for the two subsidences has practically disappeared. In 50 million years, the gradient is practically uniform down to $50\ km$. The disturbance$(\Delta T^{I} + \Delta T^{II})$). tends to zero very slowly. Even after $300\ million\ years$ the temperature down to the depth of the base of the crust, $35\ km$, is still as high as 20 percent of its initial value.

The term ΔT^{III} is given by equation 17 in terms of the function $\theta(z,t)$. The latter function represents the effect of the distortion of the substratum when the zero-temperature surface is at $\boldsymbol{z} = -\infty$, The term ΔT^{III} is shown in Figure 8 for $s = 6$ and $s = 13$ kilometer.
The heat-flow equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial^2 z^2}, \tag{38}$$

indicates that the rate of change of the temperature is approximately inversely proportional to the radius of curvature of the temperature profile, and that the sign of $\partial T/\partial t$ is such that this profile tends to become straight. This equation of one-dimensional heat flow can be compared with the equation

$$M = EI\frac{d^2y}{dz^2}, \tag{39}$$

of bending of a beam. The moment M is in a certain way equivalent to $\partial T/dt$. Also M acts in a direction that would straighten up the beam and is proportional to the radius of curvature.

The tendency of a temperature profile, in the absence of heat sources, is to become straight.
Initially, the temperature distribution $\Delta T^{III}/T_1$ presents two bends, a sharp bend at the base of the crust and a smooth one in the substratum, The smooth bend in the substratum gradually straightens up, feeding heat to the sharp bend at the base of the crust. The substratum bend becomes straight later than that at the base of the crust because its straightening involves the change in temperature of a greater volume of material. In about $50 \times 10^6\ year$, the distribution becomes practically a straight line from the surface to more than $90\ kilometer$ depth (Figure 8), *i.e.,* the surface gradient increases with time from zero to a maximum value. Afterward, the deficiency in the heat supply from the interior causes the gradient to decrease. The time at which the maximum gradient is readied can be determined by equation 19 by making $d/dt\ (\Delta \mathrm{T}^{\mathrm{III}})_{\boldsymbol{z=0}} = 0$, where by

$$t = \frac{3}{4}\frac{H(2-aH)}{a\alpha}, \tag{40}$$

If it is assumed that $a = 0.23 \times 10^{-6} cm^{-1}$ and $\alpha = 0.010\ \mathrm{cm}^2\mathrm{sec}^{-1}$, then $t = 113 \times 10^6$ yr for $H = 38 \times 10^6$cm,and $t = 141 \times 10^6$ yr for $H = 45 \times 10^5 cm$. When a is very large, i.e., when the deformation is limited to a narrow zone at the base of the crust, formula (40) reduces to

$$t = 3H^2/4\alpha. \tag{41}$$

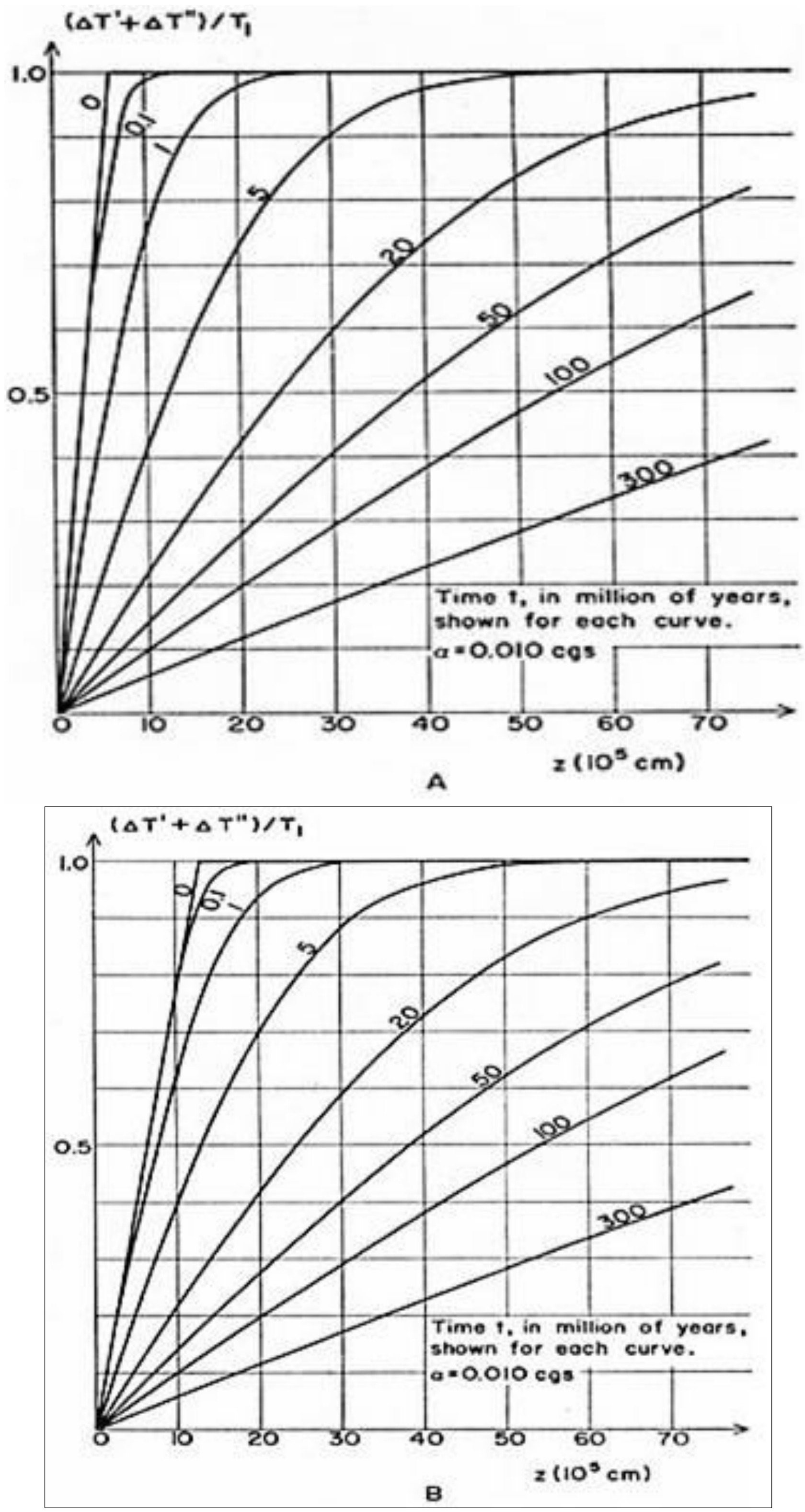

Figure 7 . Decay of Surface Disturbance ($\Delta \boldsymbol{T}^{\boldsymbol{I}} + \Delta \boldsymbol{T}^{\boldsymbol{II}}$). In A, for $= 0$, it has value 1 at $z = 6\ km$; in B, for $t = 0$, it has value 1 at z= 13 km.

It gives $\boldsymbol{t} = 48 \times 10^6$ yr for $H = 45 \times 10^5 cm$ and $\boldsymbol{t} = 34 \times 10^6$ yr for $H = 38 \times 10^5\ cm$. Hence, even if the subsidence would form a sharp front at the base of the crust, the maximum gradient would be felt at the surface only after $35 \times 10^6 6\ yr$.

The three components ΔT^I, ΔT^{II}, *and* ΔT^{III} of the total deviation ΔT_a from the final steady state have been determined. Since the temperature variation which occurs after the subsidence is of interest, the initial state is taken as datum. Hence, what is wanted is

$$\Delta \boldsymbol{T_A} = |\Delta T\_a\ (z,\ t)|\,_0^t = \Delta T_a(\mathbf{z}, t) - \Delta \boldsymbol{T} a(z, 0). \qquad (42)$$

This variation is shown in Figure 12 for the subsidence of 6 and 13 km, but it will be discussed hereafter.

Components of temperature deviation, when the heat is generated in the crust

When the heat is generated in the crust, the subsidence produces a temperature deviation, from the final steady state, which is

$$\Delta \boldsymbol{T_b} = -\Delta T_b^I(z,t) - \left|\Delta T_b^{II}(z,t)\right|_t^\infty. \qquad (43)$$

The term ΔT_b^I represents the decay of the deviation from the initial steady state, and ΔT_b^{II} is the temperature build-up produced by the new radioactive-heat sources. The distribution of sources after the subsidence and the sedimentation is considered as the sum of the initial sources and of additional sources. Thus the initial steady state can be considered as being maintained by the initial sources. The additional sources consist of two slab distributions, as shown in Figure 5. The temperature deviations are expressed in terms of T_2 as unit, which is the ultimate temperature increase in the substratum.

For long times, it is enough to considerΔT_b^{II}, because the deviation of ΔT_b^{II} from its ultimate value varies less rapidly than the decay of ΔT_b^I, namely as $t^{-1/2}$ as agains $t^{-3/2}$.For long times an asymptotic value of $\Delta T_b$is obtained, namely

$$\Delta T_b = -\frac{z}{\sqrt{\pi\alpha t}}\left(T_2^{upper} + T_2^{lower}\right), \qquad (44)$$

where T_2^{upper} and T_2^{lower} are the T_2's for the upper and lower radioactive slabs, respectively. The term ΔT_b^I is given by

$$\Delta T_b^I = T_2\{i^1\,erfc\ (z - H_0 - s/2)_1 - i^1\,erfc\ (z + H_0 + s/2)_1)\}/2(H_0 + s/2)_1 - T_2\,erfc(H_0 + s/2) - T_2\{i^1 erfc(z - s)_1 - i^1 erfc(z + s)_1 - i^1 erfc\ (z + s)_1\}/2s_1 + T_2 i^1 erfc\ s_1, \qquad (45)$$

where i' *erfc* is the first integral of the error function,$(z - H_0 - \mathbf{s}/2)_1 = (z - H_0 - \frac{s}{2})/\,(\boldsymbol{\alpha t})\mathbf{1}/\mathbf{2}$,etc.

The decay of ΔT_b^I is shown in Figure 9 for subsidences of 6 and 13 km. Initially,ΔT_b^I is a triangular temperature distribution which has value zero at $z = 0$, its maximum value at $z = s$, and is zero at $z = H_0 + s/2$. In $5 \times 10^6\ yr$ the maximum value of ΔT_b^I is reduced to less than one-third its initial value, and in $50 \times 10^6\ yr$, it has practically disappeared.

The term ΔT_b^{II} is the sum of two parts, one for each of the radioactive slabs which represent the change from the initial condition. The temperatures produced by each of these slabs are obtained from equations 27, 28, and 29. When the heat-source intensities in the sediments and in the crust are equal, *i.e.,* $q_1 = q_2$, only the lower slab is left. Table 2 gives the dimensions of the slabs for the two values of the subsidence considered.

The temperature build-up produced by the radioactivity of the slabs is shown in Figures 10 and 11. The linearity of the deviation from the steady state from about $100\ million\ years$ justifies the use of the asymptotic formula for large t. This has been used for 1000, 2000 and $3000\ million\ years$.

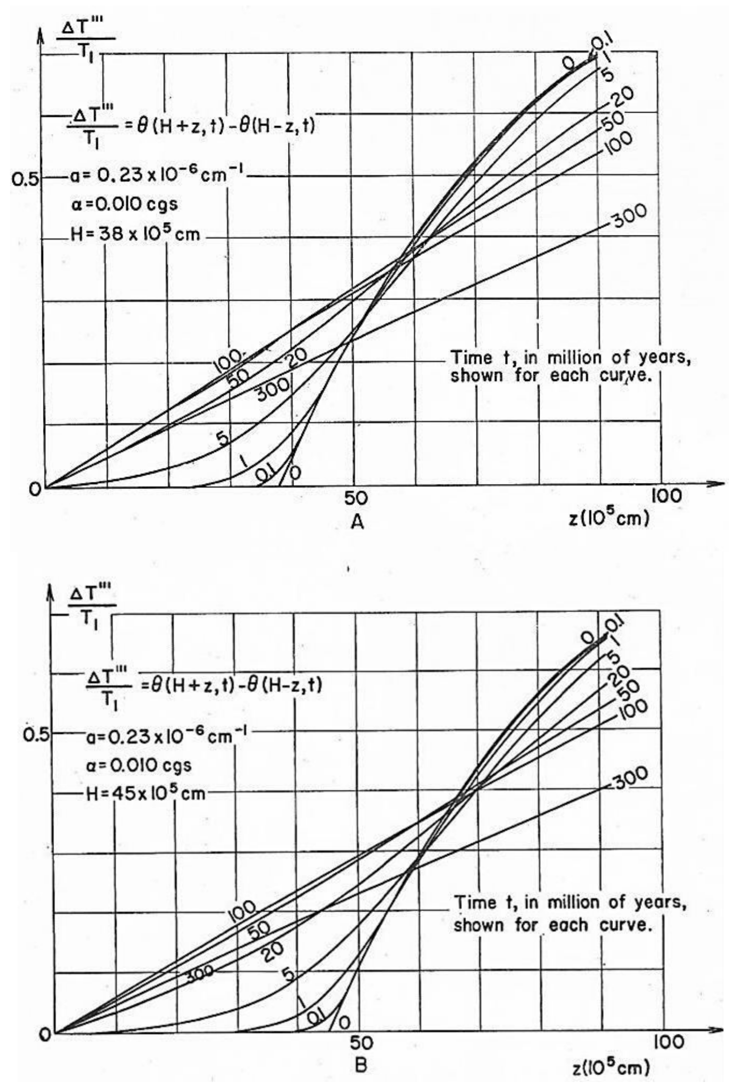

Figure 8 . Decay op the Temperature Perturbation $\Delta \boldsymbol{T}^{\boldsymbol{III}}$ Produced by Deformation of the Substratum by a Rapid Subsidence A, for a subsidence of 6 km; B, for a subsidence of 13 km.

Because of their relative proximity to the surface, the effect of the two upper slabs is felt rather rapidly at the surface. One million years is enough for the gradient at the surface to almost reach its steady-state value. In about $20 \times 10^6\ yr$, the temperatures above these slabs have practically reached the steady state. Only after 1 million years is there any perceptible increase in the temperature at the depth of the base of the crust; even after $50 \times 10^6\ yr$, this temperature is only half its steady-state value.

Because of the greater depth of the two lower slabs, it takes longer for their effect to be felt at the surface. About $5 \times 10^6\ yr$ is required for a detectable increase of the surface gradient. After about $20 \times 10^6\ yr$, the gradient is practically uniform between the surface and the top of the slab. But even after $50 \times 10^6\ yr$, it is only slightly more than half its steady-state value, and still after $300 \times 10^6\ yr$,, the deviation from the steady state is significant. The components ΔT_b^I and ΔT_b^{II} of the deviation ΔT_b from the final steady state have been obtained. To determine the variation with respect to the initial state, one takes instead. This variation is shown in Figure 13 for the subsidences of 6 and 13 km. It will be discussed hereafter.

$$\boldsymbol{\Delta T_b = |\Delta T_b(z\,,t)|_0^t = -\Delta T_b^I(z,\ t) + \Delta T_b^I(z\,,0) + \Delta T_b^{II}} \qquad (46)$$

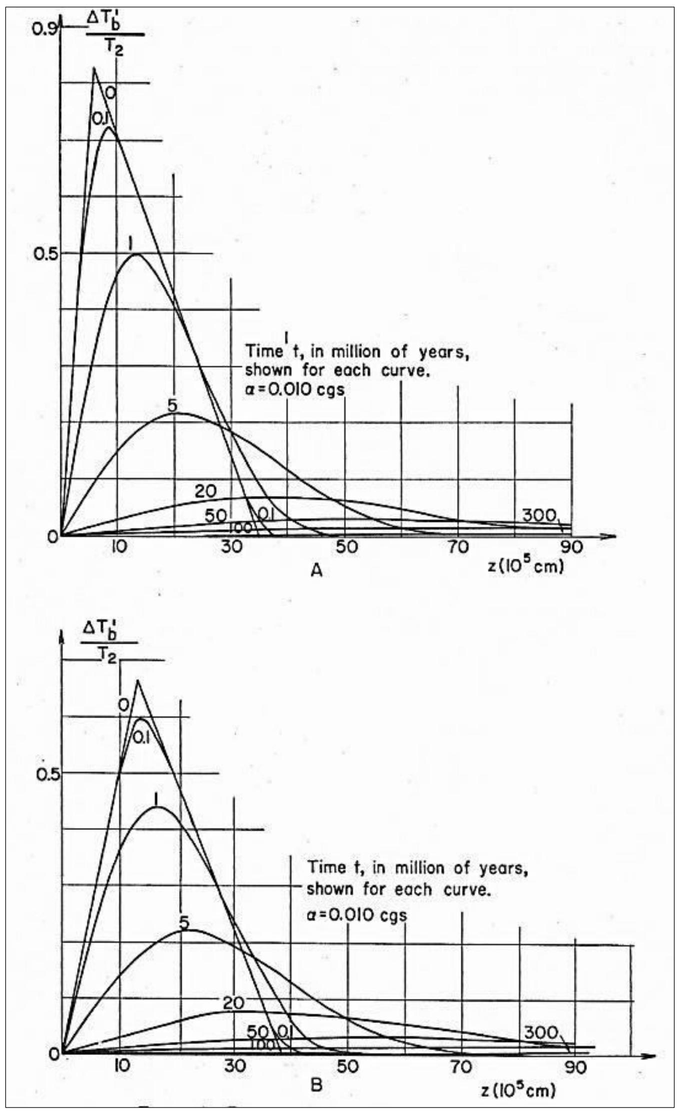

Figure 9 . Decay of Temperature Distribution $\Delta \boldsymbol{T}^{\boldsymbol{I}_b}$.In A, it initially extends from the surface down to $\mathbf{35}\ km$; in B, it initially extends from the surface down to $\mathbf{38.5}\ km$

Discussion of rapid-subsidence assumsion

The suitability of the rapid-subsidence model can be judged only if the order of magnitude of the subsidence velocity in actual basins is known. The only direct indication of subsidence throughout geologic time at present is the thickness of sediments. Present-day rates of deposition are not suitable to estimate subsidence, because they are controlled by other factors besides rate of subsidence. For the Cenozoic—an era with a fairly complete record— the average rate of deposition has been estimated to be 0.67 kilometer per million years (Schuchert,1931, p. 44). Because of erosion, compaction, and periods of no deposition, this rate is likely to be smaller than the subsidence velocity. Present rates of subsidence in some basins are larger than the figure mentioned. Kidwell and Hunt (1958), estimate that the average rate in Eastern Venezuela is 10 kilometer per million years. Moore (1955) found that the present rate is 2.1 kilometer per million years. The subsidence during one orogenic cycle is equal to at least twice the thickness of the sediments which is later found in the basin. Thus, the subsidence velocity may be of the order of 1.3 kilometer per million years. The subsidence of 6 kilometer would require about 5 million years, and that of 13 kilometer about 10 million years.

Table 2.—Dimensions op Radioactive Slabs (In 106 cm)

Dimension	Subsidence of 6 kilometer	Subsidence of 13 kilometer
h_0, upper slab	2	2
h, upper slab	8	15
H_0, lower slab	32	32
H, lower slab	38	45

The term $\alpha\,\partial^2 T/dz^2$ in the heat equation, equation 1, has been neglected in the consideration of the subsidence. The only change of temperature during rapid subsidence is produced by the displacement of the medium. The question is how large is the change due to diffusion, which occurs during the subsidence, in the initial steady state. This effect can be estimated by comparing it with the effect of diffusion upon the rapid-subsidence model after an interval of time equal to that required by the subsidence. In one case there is diffusion and subsidence, and in the other, first only subsidence and then only diffusion.

The surface perturbation is represented by the sum $(\Delta T^I + \Delta T^{II})$ in Figure 7. The surface gradient of $(\Delta T^I + \Delta T^{II})$has lost 75 per cent of its initial value in 5 million years for the subsidence of 6 km and 62 per cent in 10 $million\ years$ for the subsidence of 13 km. The times mentioned here are equal to the times required by the respective subsidences. The perturbations $(\Delta T^I + \Delta T^{II})$for the subsidences of 6 and of 13 km, expressed in terms of T_2, become practically equal after 5 million years. These times are comparable to the times required by the subsidence. Thus, these two observations indicate that the surface gradient and the temperatures in the sedimentary layer given by the rapid-subsidence model are too low. But for the subsidence velocity considered herein, 1.3 $km\ per\ million\ years$, the discrepancy with respect to the rapid-subsidence model disappears in about 5 $million\ years$.

In the model in which the heat comes from the deep interior, because of the deformation of the substratum, $\partial^2 T/dz^2 \neq 0$ in the substratum. The adjustment of this perturbation is shown in Figure 8 for the subsidences of 6 and 13 kilometer. This perturbation is practically unchanged in the zone of the substratum even after 5 and 10 million years for the 6 and 13 kilometer subsidence, respectively. Hence there is no appreciable error here because of the assumption of rapid subsidence.

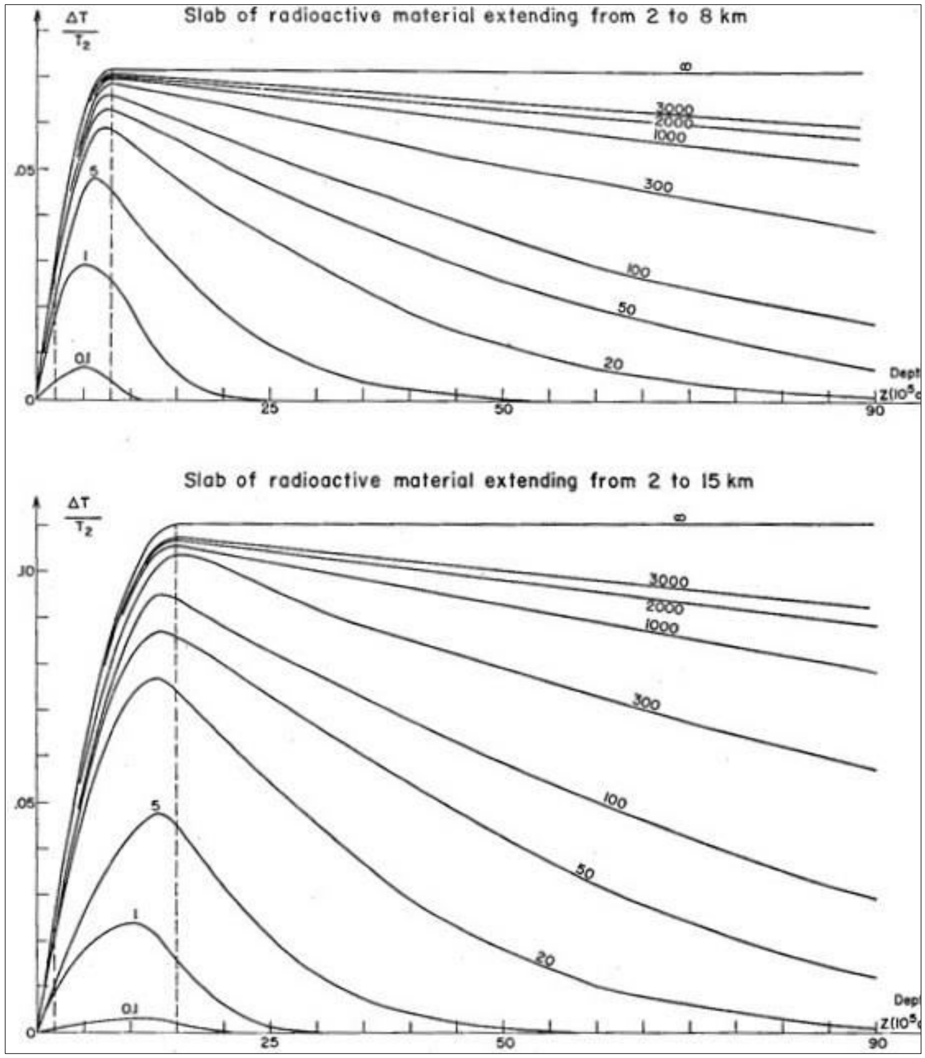

Figure 10. Temperature Build-up by Radioactive Material of Upper Slab(*Time interval after subsidence in 106 years*)

However, the effect of rapid subsidence in the lower part of the crust is yet to be considered. If the subsidence had taken place gradually, instead of instantaneously; the temperature rise in the lower part of the crust would have been smaller than shown in Figure 8. This is because the thermal gradient in the substratum builds up gradually during the subsidence. In Figure 8A, which corresponds to the 6 kilometer subsidence, the area between the curves, for $t = 5 \times 10^6\ years$ and for $\boldsymbol{t} = 0$ and which lies above the base of the crust is of interest. This area represents the heat conducted during 5million years from the substratum into the crust. After 5 million years the heat escape at the surface is still negligible. It is easy to show, from equation 15, that the excess gradient increases proportionally to the subsidence. Thus, its average value is half its ultimate value. Accordingly, the area referred to would be at most equal to half what is shown in Figure 8. Allowing for this factor, the error is greatest at the base of the crust, but even there it is not serious. For a subsidence of 6 kilometer occurring in 5 million years, the error on the temperature variation can be estimated to be less than 10 per cent of the ultimate temperature variation, and for the subsidence of 13 kilometer occurring in 10 million years to be less than 12 per cent.

When the heat is generated in the crust, the surface perturbation ΔT_b^I and the temperature build-up $\Delta \boldsymbol{T}_b^{II}$ produced by the additional radioactive slabs must be considered.

The peak value of $\Delta \boldsymbol{T}_b^I$ is reduced in about 5 and 10 million years to 26 and 24 per cent, respectively, of its initial value for the subsidences of 6 and 13 km, respectively (Figure 9). Hence, this term cannot introduce too large an error even in the sedimentary layer, where it has its greatest value. As for ΔT_b^{II}, only the lower slab need be considered, since it can be assumed that $(q_1 — q_3) \ll \boldsymbol{q_2}$. The temperature build-up produced by the lower slab is shown in Figure 11 for the subsidences of 6 and 13 kilometer. In 5 and 10 million years, respectively, the disturbance has barely reached the surface. Hence, for the subsidence velocity considered, $\Delta \boldsymbol{T}_b^{II}$ is quite suitable as given by the rapid-subsidence model. The rapid-subsidence assumption introduces practically no error in the substratum. In the lower part of the crust the maximum error is about 10 and about 12 per cent for the subsidences of 6 and 13 km, respectively. If the subsidence velocity were of the order of 1.3 $km\ per\ million\ years$, the rapid-subsidence model would give too low temperatures in the sedimentary layer for the first 5 $million\ years$. But after a time equal to that required for the subsidence, the rapid-subsidence model becomes more and more suitable. The finite subsidence velocity introduces an uncertainty in the time origin of the rapid-subsidence model. For the study of events which occur long after subsidence, the instantaneous assumption is adequate. Moreover, its effect is always unimportant in the lower part of the crust and substratum

Temperature variation after subsidence of a basin

To determine the temperature variations produced by subsidence, two different models have been investigated. In model 1 the heat comes from the deep interior; in model 2, the heat is generated in the earth's crust.

Numerical calculations were performed for two cases; namely, when the subsidence is 6 and 13 km. The results for the hypothesis that the heat comes from the deep interior are shown in Figure 12, and for the hypothesis that it is generated in the crust, in Figure 13.

A number of general observations can be made about these temperature variations after a subsidence. The temperature increases much more rapidly in the sediments than in the lower part of the crust. For instance, the temperature variations in the new layer of sediments, for the subsidence of 6 $kilometer$, have reached 74 per cent of their ultimate values after 5 $million\ years$ in both models, whereas at the base of the crust they have reached only 15 and 3 percent in models 1 and 2, respectively. A similar observation can be made for the 13 kilometer subsidence. When the heat comes from the deep interior, there is, for a certain length of time, a zone of cooling in the substratum. For instance, after 20 $million\ years$ the cooling at 90 km is about 0.08 $\boldsymbol{T_1}$. After about 20 million years, the deviation from the final equilibrium condition in the crust is a reasonably linear function of the depth. For large times, the deviation, varies as $t^{-3/2}$ when the heat comes from the interior and a $t^{-1/2}$ when it is generated in the crust.

In general, the results for the two hypotheses about the origin of the heat are similar. In the sedimentary layer and upper part of the crust, they are practically the same up to about 5 $million\ years$. But afterward the temperature increases more rapidly when the heat comes from the deep interior. The rates of increase differ more in the lower part of the crust. The temperature increases more rapidly in the model in which the heat comes from the deep interior. The contrast is even greater in the substratum.

The evolution of the temperature profiles which exist at any time after the subsidence is illustrated for the two models in Figure 14. The rapid subsidence and sedimentation sets a perturbation at the level of the original surface and another which extends into the substratum from the base of the crust. When the heat is generated in the crust and the subsidence is instantaneous, only the first of these perturbations has to be considered. The subsequent evolution of the temperature profile can be described as a gradual adjustment of this perturbation. The temperature adjustment progresses gradually from the presubsidence surface downward. When the heat comes from the deep interior, both perturbations are present, and the temperature increases more rapidly, particularly in the lower part of the crust. The times required for the adjustment of the temperature profile are large enough to be of significance in geological processes.

The increases of the temperature with time at the base of the sedimentary layer and at the base of the crust are shown in Figure 15 for the subsidences of 6 and 13 km. The continuous-line curves correspond to heat generated in the crust, and the broken-line curves to heat coming from the deep interior. Since heat may be derived from both these sources, the actual curves may he between these two curves.

An important feature of these curves is the large and rapid increase of temperature which takes place during the first 20 million years or so. Thereafter, the temperature increases at a much smaller rate. A similar observation is valid for any intermediate depth in the crust. The sharp bend in the temperature curves is more apparent in the case of heat generated in the crust and for the smaller subsidence.

The conditions represented by the continuous-line curves of Figure 15A, which correspond to the subsidence of 6 kilometer and to the heat being generated in the crust, are probably nearer to the actual conditions in a basin than for the other cases considered here.
The actual values in degrees centigrade of the temperature increase are obtained: by multiplying the relative variations, shown in Figures 12 and 13, by the factor T_1 or T_2, as the case may be. If all the heat reaching the earth's surface is derived from the radioactivity of the crust, only T_2, which is given by equation 22, is needed. The quantities which enter in this formula are taken as: $q = 4 \times 10^{-13}$ cal cm^{-3} sec^{-1}, $H = 32 \times 10^5$ cm, $k = 0.006$ cgs, and $i = 6 \times 10^5\ cm$. The values thus obtained for T_2 are indicated in the first line of Table 3.

The solution obtained for the temperature variations after subsidence is based on the assumption that the thermal conductivity is constant. However, it decreases with the temperature. If this decrease were taken into account, the temperature variations would be greater than those obtained. Assuming $\mathrm{k} = 0.008\ T_0/T$ where $T^0 = 290°\ K.$, I have calculated the ultimate temperature at the base of the crust (Grossling, 1951, PhD thesis, London Univ.). The temperature increases which result for subsidences of 6 and 13 $kilometer$ are shown in the second line of Table 3. Clearly, the decrease of conductivity with temperature leads to much greater temperature variations than would be the case if the conductivity were constant and equal to that of the surface rocks. However, in both cases the temperature variations are geologically significant.

Significance of temperature variations

Significance for lithification

The sediments of a basin gradually become lithified as a result of burial. This lithification depends mainly on the temperature and on the confining and differential stresses. Although it is not completely understood how the temperature affects the lithification, some experimental evidence indicates the importance of the temperature. Fairbairn (1950) found that the strength of quartz and quartzite decreases markedly at $150 - 315°C$. Maxwell and Verrall (1954) showed experimentally that in sands the degree of grain shattering and orientation and the degree of cementation increase with the temperature. Their results suggest that certain temperature thresholds exist below which cementation does not occur.

As for the effect of the stresses, if the sediments accumulate on a horizontal subsiding basement and the deformation in the horizontal direction is zero, one of the principal stresses is vertical, and the other two are horizontal. The state of stress on a small cube of sediments can be resolved in a hydrostatic stress plus a deviatoric stress. The normal force acting on a small plane interface is partly taken by the solid matrix of the sediment and partly by the interstitial fluid. The load on the solid matrix, when expressed as normal stress over the total area of the interface, is called contact effective stress. This is the stress directly responsible for the compression of the solid matrix. The part of the hydrostatic stress taken up by the solid matrix may be called confining stress. Obviously, the entire tangential stresses are supported by the solid matrix.

Most of the compaction of sediment occurs at shallow depths and consists of compression of the grain matrix and squeezing out of the interstitial fluids. This stage usually extends in the pressure range from 0 to about $300 psi$. Because of viscous fluid-flow resistances, the fluid pressure is greater than the hydrostatic fluid pressure corresponding to the height of the fluid column. As the compaction progresses, the excess pressure is gradually transferred to the solid matrix.

For this kind of consolidation, in a one-dimensional system, Terzaghi (1925) established the equation

$$\frac{\partial e}{\partial t} = c\frac{\partial^2 e}{\partial z^2}, \quad (47)$$

where, $e = void\ ratio\ = volume\ of\ voids/volume\ of\ solids$,
$C =$
$consolidation\ coefficient. In\ mathematical\ sense, this\ equation\ is\ the\ same\ as\ the\ one - dimensional\ heat - flow\ equation$ (equation 38).

Furthermore, the coefficients in these two equations have the same dimension, namely L^2T^{-1}. Hence, if the dimensions and boundary conditions are the same, the relative rates of change in the two processes depend on the relative value of these coefficients. For ordinary sediments, the consolidation coefficient is of the order of $(\mathbf{2} - \mathbf{10}) \times \mathbf{10^{-4}} cm^2\ sec^{-1}$, whereas the thermal diffusivity is of the order of $\mathbf{10}^{-2} cm^2\ sec^{-1}$. Hence, up to depths of a few hundred feet, the adjustment of the stresses lags behind the adjustment of the temperature, so that the progress of the consolidation depends on the adjustment of these stresses.

When the mere squeezing of the fluids has been nearly completed, the fluid pressure affects the subsequent compaction very little (Taylor and Merchant, 1940).Thereafter the grains begin to bend, crush, and shear. Resistance to further volume compression is provided mainly by the solid matrix. This stage may extend to about $\mathbf{6000}\, psi$. At larger pressures, a number of other things occur: melting at grain contacts, solidification at unstressed parts, stress-induced solution, and recrystallization. Hence, below depths of a few hundred feet, the lag in the adjustment of the confining stress is negligible. Both the stresses and the temperature of a certain

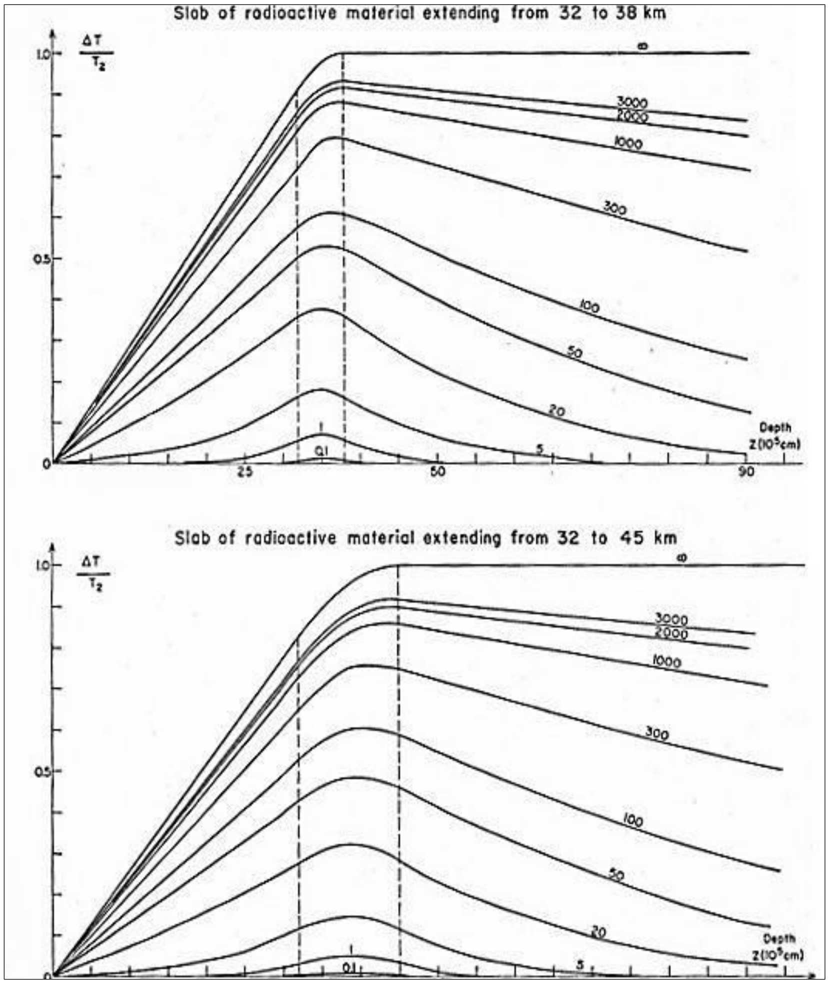

Figure 11 . Temperature Build-up by Radioactive Material op Lower Slab (*Time interval after subsidence in 106 years*)

portion of sediment increase with the subsidence. Below a few hundred feet the solid matrix is constantly subject to confining, stresses which correspond to the current depth. On the other hand, this investigation shows that the lag in the adjustment of the temperature is appreciable. Thus, the lithification may not be completed, at certain depths, because the temperature is still too far below its steady value. This seems to be corroborated by the existence of soft sediments at relatively great depths in basins such as the Gulf Coast This paper has shown that in the sedimentary layer, 70-80 per cent of the ultimate temperature increase occurs within the first 15 million years after the subsidence. It seems likely therefore, that lithification is completed during that time interval.

Significance for evolution of basins

A basin begins by subsidence of the earth's crust along an elongated trough. Sometime later comes a period of increased compression. This is the folding stage, in which the sediments and the crust are folded or fractured, or both. During this stage, probably no true rupture of the crustal material occurs, but rather the rate of shearing of the crustal rocks increases markedly. Under differential stresses, the crustal rocks can flow continuously, although at a very small rate. This flow can be described in terms of a solid viscosity. Whatever the ultimate cause of the crustal compression, the forces responsible for it work against the compressional strength of the crust. Therefore, crustal strength must be a relevant factor in the evolution of a basin.

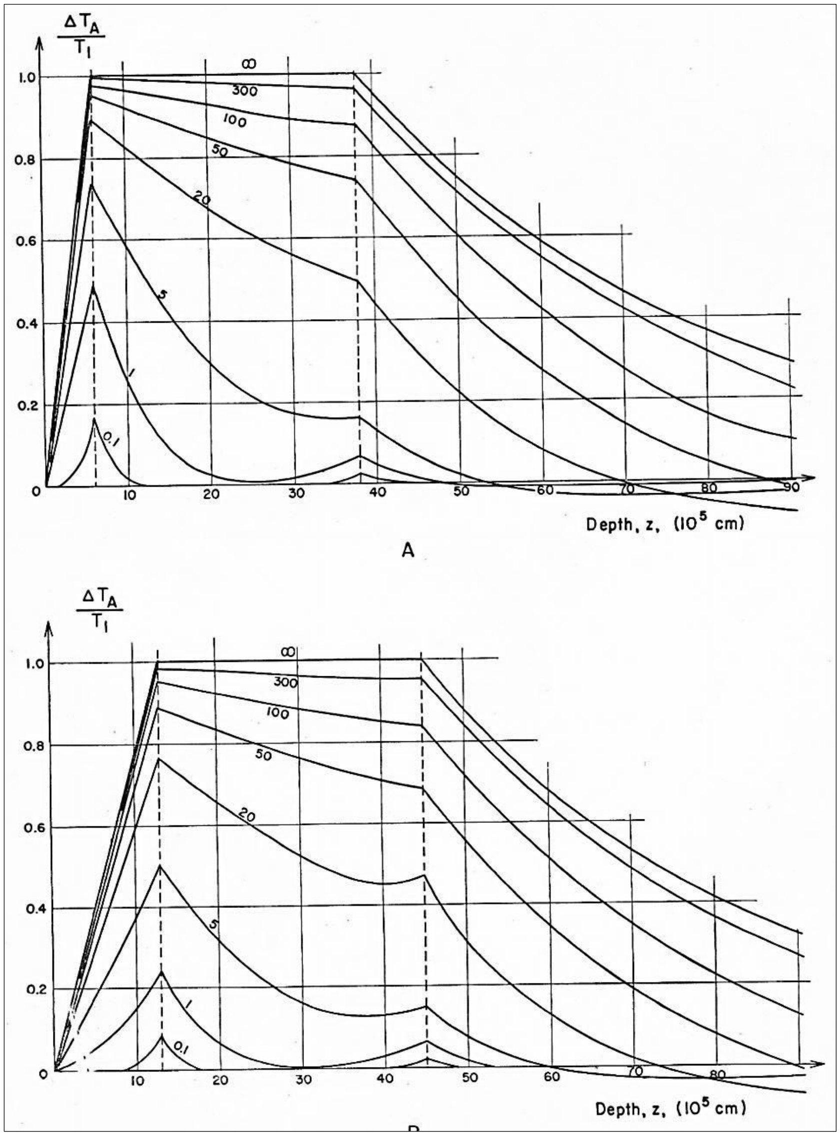

Figure 12 . Temperature Increase after Rapid Subsidence and Sedimentation when the Heat Comes from the Earth's Deep Interior.A, for a subsidence of $\mathbf{6}\,kilometer$; B, for a subsidence of $\mathbf{13}\,kilometer$. Thickness of crust is equal to $\mathbf{30}\,kilometer$, initial thickness of sediments is equal to $\mathbf{2}\,kilometer$. Deformation of substratum assumed to decrease to one-tenth at $\mathbf{100}\,kilometer$ below the crust. Time intervals after subsidence in $\mathbf{10^6}\,years$. Ultimate temperature in the crust taken as unity.

Of particular interest is whether the position of the lower boundary of the crust, the Mohorovicic discontinuity, depends on the temperature. The crustal viscosity appears to decrease markedly at the depth

where the temperature reaches a certain value. The value of this temperature is not needed herein; it is sufficient to assume that it is the temperature at the base of an undisturbed crust.

The solid viscosity of the crustal material is assumed to decrease when the temperature reaches the value it has at the base of an undisturbed crust. In consequence, the compressional strength of the crust will vary as the thickness of the crust which is at temperature smaller than that value. The temperature increase, which occurs after the subsidence, progressively reduces the crustal strength by decreasing the crustal thickness.

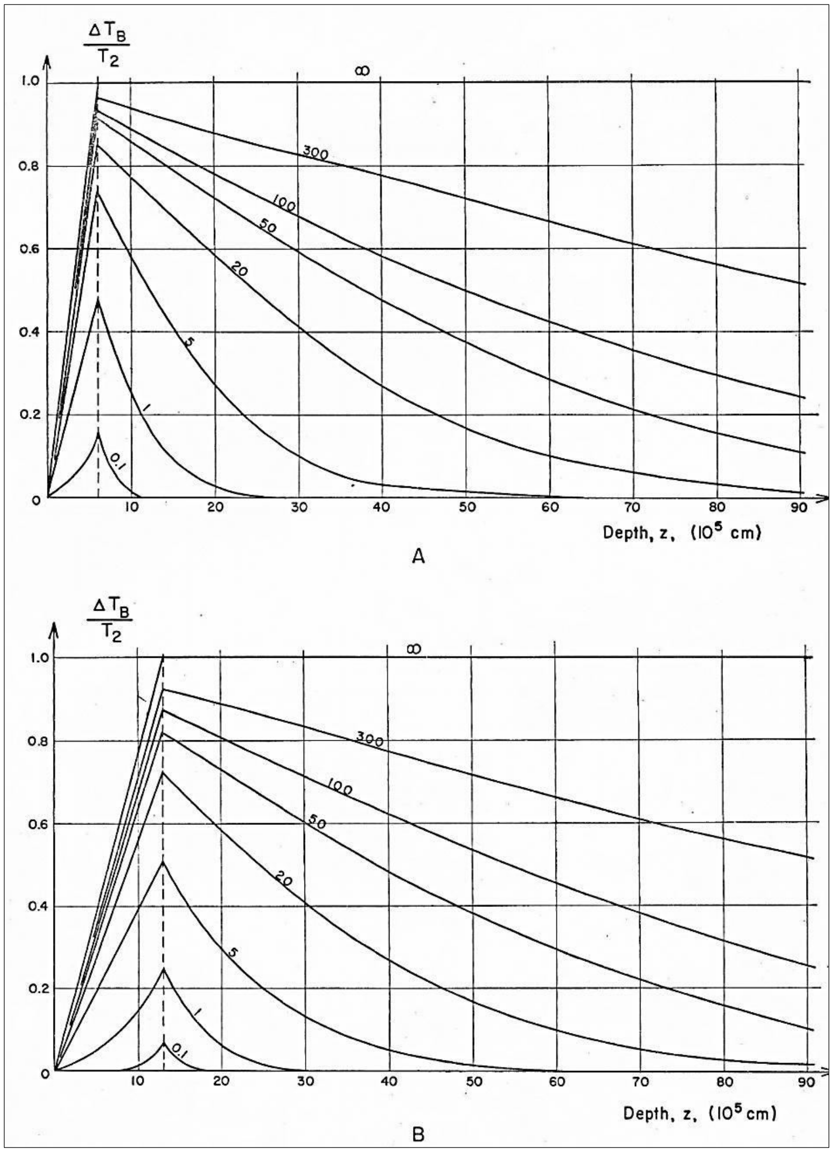

Figure 13 . Temperature Increase after Rapid Subsidence and Sedimentation when the Heat is Produced by the Radioactivity of the Crust and Sedimentary Layer. A, for a subsidence of **6** kilometer; **B,** for a subsidence of $\mathbf{13}$ km.Thickness of crust is equal to 30 km, initial thickness of sediments is equal to 2 $\boldsymbol{km}$**,** $\boldsymbol{q^1} = q^2$. Time intervals after subsidence i**n** 10^6 $\boldsymbol{years}$**.** Ultimate temperature increase in the crust taken as unity.

Table 3 -ultimate temperature increase T_2 at base of crust

	For subsidence $\mathbf{6} \times \mathbf{10^5}$ cm	For subsidence $\mathbf{13} \times \mathbf{10^5}$ cm
$k = \mathbf{0.006}$ cgs	$\mathbf{140°C}$	$\mathbf{333°C}$
$k = \mathbf{0.008}$ T_0/T cgs	$\mathbf{289°C}$	$\mathbf{907°C}$

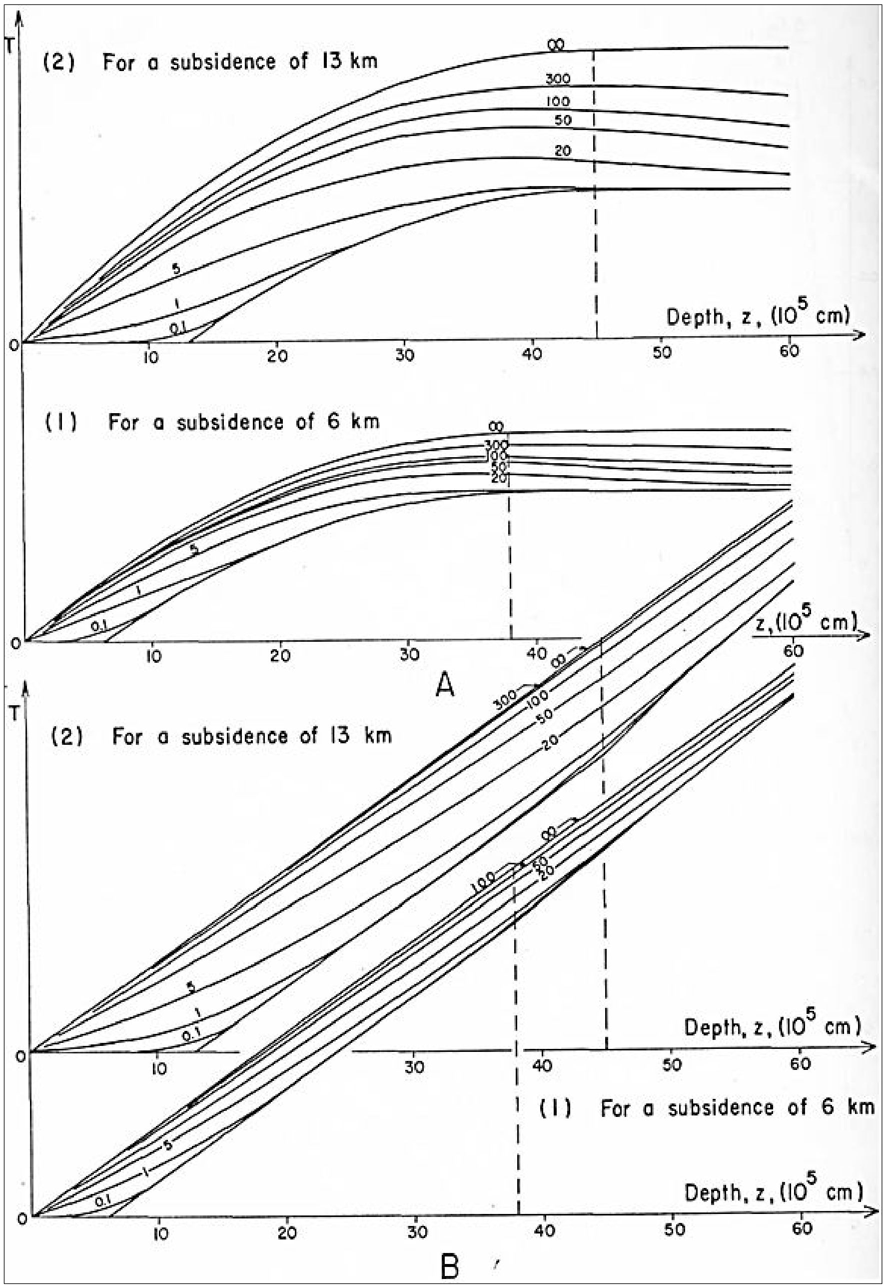

Figure 14 . Evolution of the Temperature Profile after Rapid Subsidence and Sedimentation. In **A,** the heat **is** produced by the radioactivity of the crust and sediments; in B, the heat comes from the earth's deep interior.

At the start, there is assumed to be a homogeneous and isotropic crust, of constant thickness, parallel to the geoids, and under no tectonic stress. Thus, one of the principal stresses is vertical and the other two are equal and horizontal. These stresses are assumed to vary linearly with depth. Whether the vertical stress is the maximum or the minimum or whether the three are equal depends on how the system came into being and also on the long-time strength of the crustal materials. If this stress system has acted for a long time, then the absolute values of the three stress differences are equal to or smaller than twice the yield shear stress. Tresca's maximum-shear yield criterion has been assumed for simplicity.

It is assumed now that a tectonic force, $Q,$ per unit length is applied in the x direction (Figure 16). If the part of the crust subject to this force is a long belt, such as a basin, there is no strain relief in the direction of the long axis of the belt. For the tectonic force Q to produce plastic deformation in planes perpendicular to the axis, the difference of principal stresses in this plane must be equal or larger than twice the yield shear stress.

Smoluchowski (1909) has shown that the crust cannot be buckled elastically, because the buckling compressional stress is much larger than the elastic limit. When the maximum differential stress equals the yield limit, the crust thickens plastically. Vening-Meinesz and Heiskanen (1958, p. 326-343), following Bijlaard (1935; 1936), show that the crust buckles as a result.

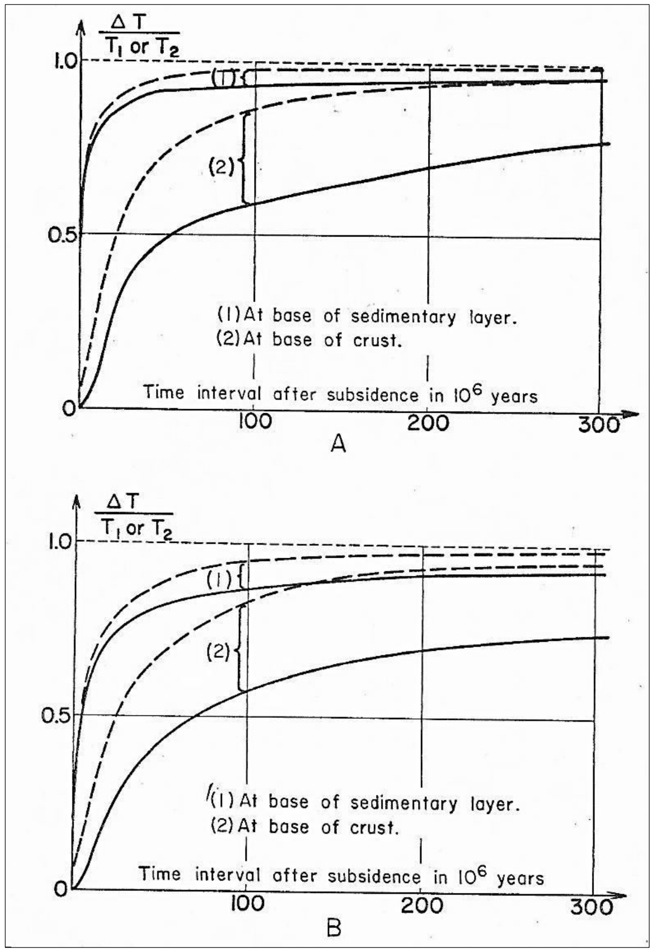

Figure 15 . Increase of Temperature after a Rapid Subsidence. A, for a subsidence of $\mathbf{6}\,km$; B for a subsidence of $\mathbf{13}\,km$. Continuous-line curves correspond to heat generated m the crust, and broken-line curves to heat coming from the deep interior.

It will be supposed that a part of the crust has begun to down buckle; therefore, the tectonic force Q is applied above the middle plane of the crust, as shown in Figure 16. Such a force is equivalent to a force of magnitude Q acting on the middle plane, plus a bending moment Q_a The stresses produced by the bending moment contribute to accentuate the bending of the crust, whereas the force on the middle

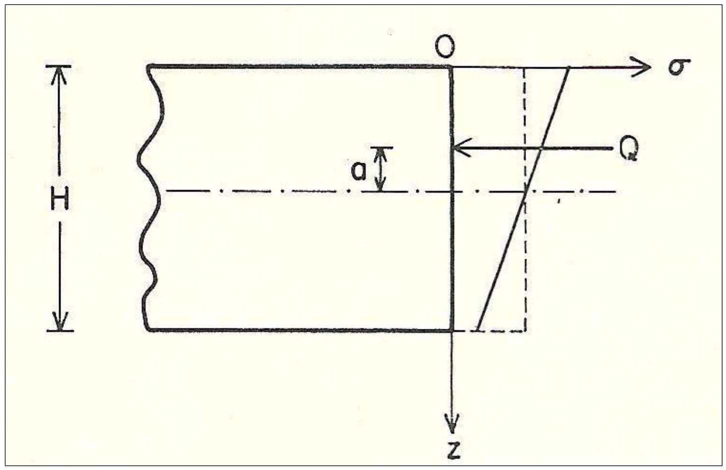

Figure 16 . Stress Distribution in the Crust Produced by ***A*** compressional force ***Q***.

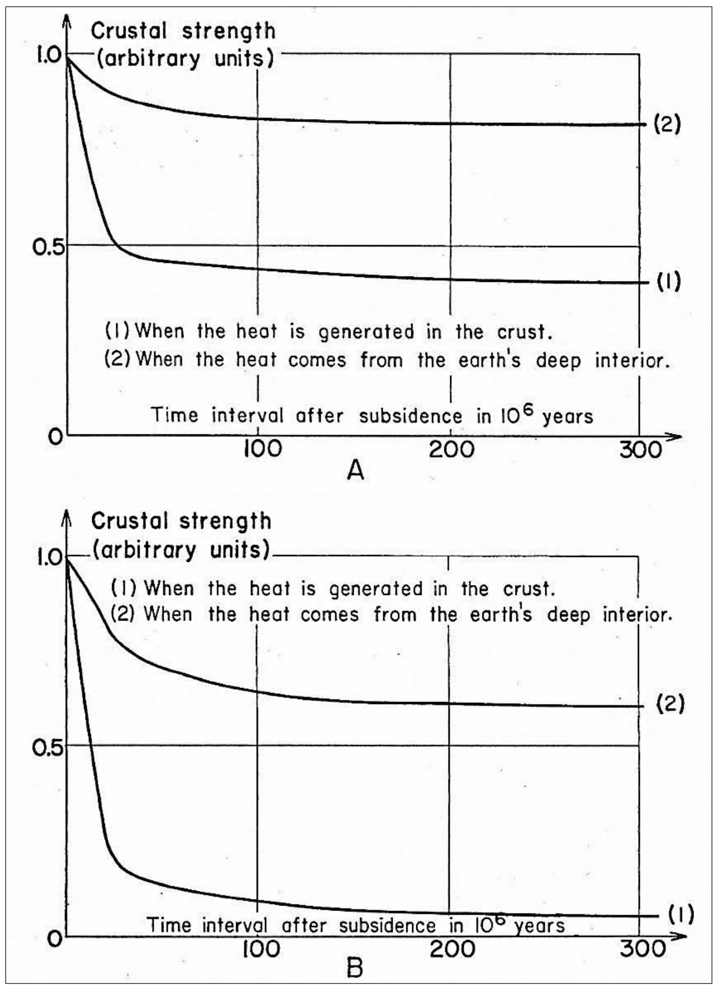

Figure 17 Variation of the Crustal Strength Because of the Increase of Temperature which Follows a Rapid Subsidence A, for a subsidence of 6 km; B, for a subsidence of 13 km.

plane merely thickens the crust without bending. The bending stress $\boldsymbol{\sigma}$ depth z is

$$\sigma = \frac{12Qa(\frac{H}{2}-z)}{EH^3}, \tag{48}$$

where
$E =$
$Young's\ modulus\ and\ the\ other\ symbols\ designate\ the\ quantities\ shown\ in$ Figure 16.
The deformation rate $\dot{\epsilon}_x$ is

$$\dot{\epsilon}_x = \frac{\boldsymbol{\sigma}}{2\boldsymbol{\eta}} = \frac{\boldsymbol{6Qa(\frac{H}{2}-z)}}{\boldsymbol{E\eta H^3}}, \tag{49}$$

where $\boldsymbol{\eta}$ is the solid viscosity.The rate of the relative angular rotation, $\dot{\gamma}$, of two sections a unit distance apart is

$$\dot{\gamma} = \frac{6Qa}{E\eta H^3}. \tag{50}$$

The progressive decrease of crustal strength, assuming that the strength varies as the inverse power of the thickness, is shown in Figure 17 for the subsidences of 6 and 13 km and for the two hypotheses about the nature of the heat. The several curves of variation of the crustal strength with time exhibit, at a certain time, well-defined bends. The strength decreases relatively quickly up to about $20\ million\ years$ and thereafter much more slowly. This observation is valid whether the heat comes from the deep interior or is generated in the crust. But the bend is more pronounced if the heat is generated in the crust. Moreover, the bends would be even more pronounced if the relative rate of angular rotation y (equation 50) had been considered. This varies with the inverse third power of the thickness.

Another thermal effect of the subsidence which may affect the evolution of a basin is the temporary cooling of the substratum if the heat comes from the deep interior. During the subsidence, a given layer is displaced to greater depths, practically maintaining its temperature. Thereafter, its temperature actually decreases for a certain length of time because it conducts more heat to shallower layers than it receives from deeper ones. This zone of cooling in the substratum may effectively lock, for a certain time, the subsequent uplift of the crust.

Conclusions

Subsidence and sedimentation produce, important temperature variations in the earth's outer layers. Similar results are obtained for the two hypotheses as to the origin of the heat: (1) heat comes from the deep interior, and (2) heat is generated in the crust. The variations are practically the same up to about 5 million years in the sedimentary layer and upper part of the crust. After about 5 million years, the increase becomes more rapid when the heat comes from the deep interior. But in the lower part of the crust, when the heat comes from the deep interior, the temperatures are built up more rapidly from the beginning. During the first $20\ million\ years$ or so, the temperature increases rapidly; thereafter the rate of increase is much smaller. For large times, the deviation from the steady state is a linear function of the depth and varies as $t^{-3/2}$ if the heat comes from the deep interior and as $t^{-1/2}$ if it is generated in the crust.

The times required for the adjustment of the temperature are large enough to be of significance in geological processes. Thus, the temperature variations here determined should interest other investigators.

The gradual increase of temperature after subsidence should affect the lithification of the sediments and should decrease the crustal strength. Lithification is likely to be completed within the first $15\ million\ years$ after the subsidence. The crustal strength decreases relatively quickly up to about $20\ million\ years$ and thereafter much more slowly. This decrease of crustal strength after the subsiding stage in the evolution of a basin may control the time at which the folding stage begins, thus acting as a trigger effect. The folding stage would take place $15 - 20\ million\ years$ after the subsidence. When heat comes from the deep interior, the temperature in the substratum below a certain depth, which varies with the temperature, actually decreases for a time. This cooling may retard the subsequent uplift of the basin by increasing the solid viscosity of the substratum.

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