The Magnitude of strain in the Pachmarhi

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ABSTRACT − A semigraphical method of analysis of large strain based on Nadai's strain components and utilizing a Mohr construction is outlined for problems of interest in structural geology. Finite homogeneous strain theory is applicable to meas-

urement and analysis of strains from geologic features small enough to be included within regions of homogeneous strain. Use of this theory and of the strain ellipsoid and its properties implies nothing about isotropv or homogeneity of the rocks or about the stress-strain relation during deformation, and therefore its application is valid over a much wider range of phenomena than most geologists realize.Strain can be measured, and under favorable conditions the principal strain axes can be determined, by using not only the familiar ooids, pebbles, and crinoids columnals but also a large group of fossil remains and impressions and primary sedimentary structures. The most useful features are single linear elements in which extension can be measured; the next most useful are groups of three directions of known original relative orientation, and the least useful are originally perpendicular line elements. Under ideal and unusual conditions volume change can be measured. Examples of strain analysis are worked out in detail to illustrate both the versatility and limitations of the method.

Introduction
General statement

The motion imparted to rocks during mountain building can be separated into two parts: (1) translation and rotation of the mass as a whole and (2) relative motion of parts of the mass with respect to one another. Such relative motion is called here the strain. Although both parts are in general combined in the deformation observed by the structural geologist, in special cases there may be no translation, or no strain. Where strain has been appreciable, we say that the rocks have "flowed."

There is usually no way to determine (1) from field evidence, although in many cases the amount of rotation of bedded rocks can be estimated. There are ways of determining the strain quantitatively. This paper is concerned with one method of determining the magnitude of large strain in deformed rocks.

There are several reasons why the study of strain is important in structural geology. A statement of the strain is the most precise and fundamental description of how a rock has deformed. Strain is a quantitative measure of the amount of flow. Detailed knowledge of strain in structures such as folds may lead to a clearer understanding of their origin. Knowledge of strain may also make it possible to deduce the stresses responsible for deformation. Attempts have been made to estimate stress by deforming models of observed structures and by petro-fabric analysis. Direction but not magnitude of stress is estimated. Another approach is to treat the rock as a plastic solid. A stress-strain law determined in the laboratory might be applied to the strain measured in the rock to determine the stress responsible for the deformation. Although no stress-strain law valid for large deformations is yet known, it appears worthwhile to explore the first step of this approach, the determination of strain.

Knowledge of strain may help to identify the flow mechanism responsible for the deformation of a particular group of rocks. Discontinuity of strain over short distances might suggest, for example, that shear fracturing and rotation of fragments were involved.

Although strain was measured in deformed rocks very early in geology, it has never received a great deal of attention. Features whose original shape is known in sufficient detail are scarce and rather complicated calculations are required to compute strain for all but the simplest situations. This paper gives an alternative semi graphical method of determining strain, based on Nadai's work (1950) and shows its application to geologic problems. Many of the characteristics of the strained state are more readily grasped using this method than through formulae, and it is hoped that this clarification may renew interest in geologic strain measurement and in the discovery of features, perhaps more common than suspected, which might yield measurable strain.

Mathematical descriptions of strain

Mathematical descriptions of strain fall into one of two groups, depending on the magnitude of the strain. The one describing small strains, the infinitesimal strain theory, is the simpler of the two. However, it cannot be used for the large strains of interest in structural geology.

The simplest large or finite-strain description is that of finite homogeneous strain. However, most geologic strain taken to include a large volume of material, such as an entire fold, is not homogeneous. Such strain can be treated, however, by analyzing one at a time small volumes of material, such as a small part of the limb of a fold.

All that is needed is that displacements are continuous and that strain varies gradually. Then at least locally the equations of finite homogeneous strain will apply. Many minor structures indicate that this is true; for example, many circular objects acquire an approximately elliptical shape during deformation. Faulted rock must be excluded, because a fault is an abrupt change in displacement.

We are concerned, then, with application of finite homogeneous strain theory to highly strained rocks, and in particular, with methods for measuring strain within regions of homogeneous strain. The mapping of strain and its variation in a particular structure, like a fold, is a field problem and is not considered here. The question of whether the measured strain is the total or only a fraction of the total experienced by the rock can be answered only for specific geologic situations. Breddin (1956 a, P.231-233) among others discusses various aspects of this important and difficult question.

Definitions

Strain is the change in relative configuration of the parts of a body. It is natural to use as measures of Strain The Change in length of Lines and the change in angle between a pair of lines or a line and a plane. These Determine The Quantities Extension And Shear, Respectively.

An extension is the ratio of the increase of length of a line to the original length. A shear strain is usually defined for a pair of originally perpendicular lines. In figure La such lines are a and b before strain perpendicular lines. In Figure la such lines are, a and b before strain and ar and br (Figure 1b) after strain.
The shear strain is usually defined as some function of the angle , where ψ is the amount which the line has been "Sheared" out of its original position with respect to line b.

A physical description of finite homogeneous strain is embodied in the following statements:

1. Straight lines in the unstrained state remain straight after strain.
2. Parallel lines in the unstrained state remain parallel after strain.
3. All lines in the same direction in the body undergo the same extension.
4. A sphere imagined to be imbedded in the unstrained material at any point becomes an ellipsoid after strain. The ellipsoid is called the strain ellipsoid, and its axes are defined as the principal axes of strain, or simply the strain axes. Planes that include two of these axes are principal planes. In the direction of the principal axes the shear is zero, and the extension assumes it's greatest, least, and intermediate values, the principal strains.
5. A particular ellipsoid in the unstrained state becomes a sphere after strain; it is called the reciprocal strain ellipsoid. The directions of the axes of the reciprocal strain ellipsoid are the original directions of the principal strain axes in the unstrained state.
6. In pure strain there is no translation and rotation of the body as a whole with respect to the strain axes. Therefore the directions of the strain axes before and after strain coincide, and the directions of the axes of the strain ellipsoid and reciprocal strain ellipsoid coincide. For general strain, which is not pure, the directions of the strain axes constantly change with respect to the body during deformation.
7. Only three mutually perpendicular directions in the unstrained state remain mutually perpendicular after strain; these are the principal strain axes and therefore the axes of the ellipsoids referred to in (4) and (5).

A state of strain is defined most simply by the orientation and magnitude of the three principal strains.
Analysis of strain consists either of finding the extension and shear when principal strains are known, or of finding the principal strains when the extension and shear are known for various directions. The second type of analysis is the more common in geology.

Previous measurement of strain in geology

The present concept of strain was apparently understood by Cauchy in 1822 (Love, 1944, P.8). The first geologic applications were made by Sorby (1853) and Haughton (1856). Thomson and Tait (1879) gave the first comprehensive mathematical treatment.

Strain measurement in geology was almost ignored from about 1900 until recently. One reason may have been the appearance of Becker's theories of rupture (1893) and of the formation of cleavage (1904; 1907) which were based on finite homogeneous strain and on the strain ellipsoid. These theories were not generally accepted and were eventually discarded after much discussion. They had become so irrevocably linked with finite strain that the study of finite strain was discarded also.

Quantitative strain measurement is closely tied to both petro fabric analysis and certain proposed laws of fracture and flow. Griggs (1935) discusses the fracture law of Becker, and Turner (1948, p. 16M63) discusses the difference in approach of petro fabric analysis and the study of strain.

Measurements of strain have been made by means of fossils, plastically deformed crystals (Schmidt, 1927, p. 60; Hand in and Griggs, 1951), and various features such as pebbles (Oftedahl, 1948) and ooids.

The first measurements of strain using fossils, in which either per cent elongation or the ratio of principal strains was reported, were made by Haughton (1856), Dufet (1875), and Wettstein (1886). Ernst Cloos (1947) and Rutsch (1949) give excellent reviews of this early work.

By far the most extensive studies of strain in deformed rocks are those of Haughton, Wettstein, Ernst Cloos, and Breddin. Haughton (1856) computed ratios of the three principal strains from observations of brachiopods in which ratios of certain lengths in strained and unstrained state could be compared. He assumed homogeneous strain and assumed that two principal strains lay in the plane of cleavage, one parallel with the intersection of bedding and cleavage. This enabled him to determine ratios of three principal strains from only two deformed shells.

Wettstein (1886) described the distorted fish remains from Tertiary slates beneath the Glarner overthrust. He computed from the distortion of originally perpendicular parts of skeletons the ratio of the principal strain axes in the bedding planes. He assumed that the bedding contained two principal strains, one parallel with a certain prominent lineation.

Ernst Cloos computed the magnitudes and directions of principal strains from the distortion of ooids (1947). He assumed that ooids and crinoid columnals were originally spherical and circular respectively and that their volume had not changed during deformation.

Breddin (1956 a; 1956 b; 1957) used crinoid columnals, cephalopods, pelecypods, and brachiopods to derive the direction and magnitudes of the three principal strains. To compute these quantities he assumed that: (1) the strain was homogeneous; (2) volume change during deformation had certain arbitrary values depending on the type of rock containing the fossils; (3) cleavage was perpendicular to the maximum shortening; and (4) a certain lineation in the rocks was parallel with one strain axis.

A general feature of all previous work is the assumption of volume change, of shape or orientation of strain ellipsoid, or even of the magnitude of one of the strain axes. Clearly, this is part of the information we are seeking. It is therefore of particular interest to explore ways of determining strain axes and even volume change without arbitrary assumptions or at least to determine systematically the minimum number of assumptions required to make the problem tractable.

Mohr diagrams in the analysis of strain

Choice and definition of strain components

Hershey (1952) gives several ways of describing mathematically the change of length of a line, and the shear of a pair of lines, or a line and a plane. The choice must be guided by the nature of the information available in geology and by the kind of strain analysis intended. The following were considered:

(1) Magnitude and direction of principal strains are desired.

(2) In deformed rocks, the strained state is usually known, as well as relative lengths and directions in the unstrained state. The angle between a particular line in strained and unstrained states is usually not known.

(3) In many cases bits of information must be combined. The particular definition of change of length and shear selected should have a simple physical meaning and be easily measurable for geologic features.

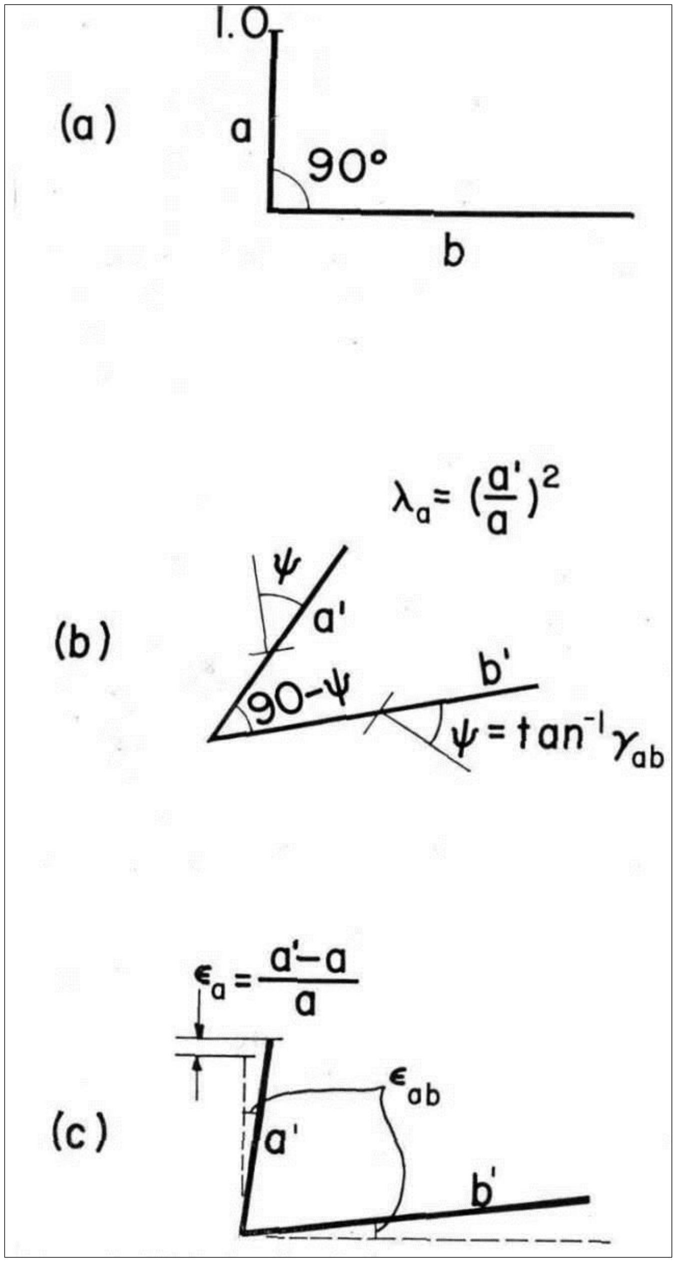

Figure 1 . Unit shear and quadratic elongation. Two originally perpendicular lines, a and b (a), are deformed in finite homogeneous strain to a' and b' (b). The unit shear of the lines, γ_{ab} and the quadratic elongation,λ_a, of the line a are shown. For comparison with components of infinitesimal strain the shear ϵ_{ab} of the two lines is given with the extension, ϵ_a , of the line a (c).

The components that best meet the above requirements and have the great advantage of graphical representation are the quadratic elongation λ and the unit shear γ (Nadai, 1950, P. 117). The quadratic elongation is the square of the ratio of a length, a' , of a line in the strained state to the length, *a,* in the unstrained state (Figure 1).

The unit shear of a line is defined as follows: During finite homogeneous strain a sphere imagined to be imbedded in the material is deformed into an ellipsoid. Any line in the unstrained state can be imagined to pass through the center of the sphere and to intersect the surface of the sphere at a point. The line is perpendicular to the tangent plane to the sphere at that point. After strain, the line and the tangent plane to the resulting ellipsoid will no longer be perpendicular, as both, in general, will be sheared out of their original positions. The unit shear, γ, of the line is the tangent of the angle ψ between the line in the strained state and the normal to the tangent plane of the strain ellipsoid. It is called "unit" shear because it is the distance a pair of tangent planes originally a unit distance apart are displaced in shear.

For the special case of strain in one of the principal planes of the strain ellipsoid the unit shear has a simpler meaning. As shown in Appendix 2 it is the tangent of the angle, ψ (Figure lb), where $(90— \psi)$ is the angle in the strained state between two lines originally perpendicular in the unstrained state. For comparison of λ and γ with the more familiar strain components of infinitesimal strain theory the latter are shown in Figure 1c.

The principal strains are the principal quadratic elongations, λ_1 , λ_2 , λ_3 in which $\lambda_1 \geq \lambda_2 \geq \lambda_3$. Zero extensional strain means quadratic elongation of unity.

Volumetric strain is found as follows: Consider a cube of edge length c with edges aligned parallel with the strain axes. Original volume of the cube is c^3. The strained cube has edges $\sqrt{\lambda_1}$,$c\sqrt{\lambda_2}$,$c\sqrt{\lambda_3}$ and a volume of $c^3\sqrt{\lambda_1\lambda_2\lambda_3}$. The change of *volume per unit volume*, or *unit volume change* is

$$\frac{c^3\sqrt{\lambda_1\lambda_2\lambda_3}-c^3}{c^3} = \sqrt{\lambda_1\lambda_2\lambda_3} - 1,$$

Mohr diagrams for plane strain

Analysis of strain involves either finding the strain components, λ and γ, for particular directions when the principal strains are given, or finding the principal strains when λ and γ are known for various directions. The second type of analysis is of greater geologic interest but more difficult than the first. In order to simplify the introductory material below it is assumed that strain components are sought for a known state of strain; the geologically more relevant problem is treated in later sections.

One of the great advantages of using the strain components λ and γ is that graphical representation not only eases the solution of many geologic problems but enables many of the properties of the strained state to be clearly visualized. Nadai showed that the equations of finite homogeneous strain when written in terms of λ and γ could be represented graphically in a diagram similar to the Mohr circle for stress or for infinitesimal strain (Jaeger, 1956, p. 9, 43). Such diagrams for finite strain are called *Mohr diagrams* - "circle" would be confusing, as they may be either ellipses or circles. Analysis of strain using Mohr diagrams becomes, by analogy with stress, either a matter of finding strain components for particular directions, given a Mohr diagram representing a state of strain, or constructing a Mohr diagram from known strain components for various directions.

A general state of strain involves relative motion in three dimensions. For simplicity strain in two dimensions is first discussed; this can then be extended to three dimensions. The particular type of two-dimensional strain treated is plane strain in which the principal quadratic elongation normal to the plane of interest, a principal plane, has the value of unity.

During finite strain a line may change direction with respect to strain axes. The direction of a line can therefore be described either in terms of position before strain or after strain. This leads to two sets of equations describing how strain components vary in direction for a given strain, referred either to strained or to unstrained states. There are two corresponding Mohr diagrams - there is only one for infinitesimal strain, for lines change direction by a negligible amount. In geologic problems the two Mohr diagrams are used either singly or together, depending on the nature of the information available.

unstrained state - Consider a state of strain defined by certain principal quadratic elongationsλ_1, λ_2,λ_3. The x and y of an arbitrary line are given by Jaeger (1956, p. 35, 36, equations 5, 18):

$$\lambda = \lambda_1 a_1^2 + \lambda_2 a_2^2 + \lambda_3 a_3^2, \tag{1}$$

$$\gamma^2 = {}^1\!/_{\lambda_1\lambda_2\lambda_3}\{\lambda_3(\lambda_1 - \lambda_2)^2 a_1^2 a_2^2 + \lambda_1(\lambda_2 - \lambda_3)^2 a_2^2 a_3^2 + \lambda_2(\lambda_3 - \lambda_1)^2 a_3^2 a_1^2\}\text{'} \tag{2}$$

where a_1, a_2 and a_3 are directions cosines of the arbitrary line referred to the strain axes before strain. For plane strain consider the $\lambda_1\lambda_3$ plane, and let λ_2 equal 1.Then the direction cosine a_2 becomes zero and these equations become

$$\lambda = \lambda_1 a_1^2 + \lambda_3 a_3^2 = \frac{\lambda_1+\lambda_3}{2} + \frac{\lambda_1-\lambda_3}{2}\cos 2\emptyset_2, \tag{3}$$

$$\gamma = \frac{1}{\sqrt{\lambda_1\lambda_3}}(\lambda_3 - \lambda_1)a_3 a_1 = -\frac{\lambda_1-\lambda_2}{2\sqrt{\lambda_1\lambda_3}}\sin 2\emptyset_1, \tag{4}$$

where $\emptyset_1$ is the angle between the arbitrary line and the λ_1 axis before strain. These expressions are analogous to those for stress in two dimensions with the exception of the term $\sqrt{\lambda_1\lambda_3}$ which is related to area change in the $\lambda_1\lambda_3$ plane. These equations define an ellipse in the Mohr diagram when λ is the abscissa and γ the ordinate (Figure 2). This is called here the Mohr diagram for the unstrained state. In a region of homogeneous strain defined by λ_1 and λ_3 the strain of any line, whose inclination to the direction of λ_1 is $\emptyset_1$ is given by a point (λ,γ) on the ellipse. The point is found by laying off a line at an angle of $2\emptyset_1$ to the λ axis and dropping down to the ellipse from the intersection of the line with a circle through λ_1 and λ_3.

The area change in the $\lambda_1\ \lambda_3$ plane per unit area is $\sqrt{\lambda_1\ \lambda_3}$— 1. This is also equal to the volume change here, as λ_2 equals 1. For the special case of no area change, $\sqrt{\lambda_1\lambda_3} = 1$, and the ellipse becomes a circle (Figure 3).

strained state - In the Mohr diagram for the unstrained state above, directions are those in the unstrained body. Alternatively a Mohr diagram can be constructed to describe the variation of strain in the strained state. The variation of the strain with direction is now given in the most convenient form by introducing reciprocal values of strain components (Jaeger, 1956, P. 37, equations 23, 24):

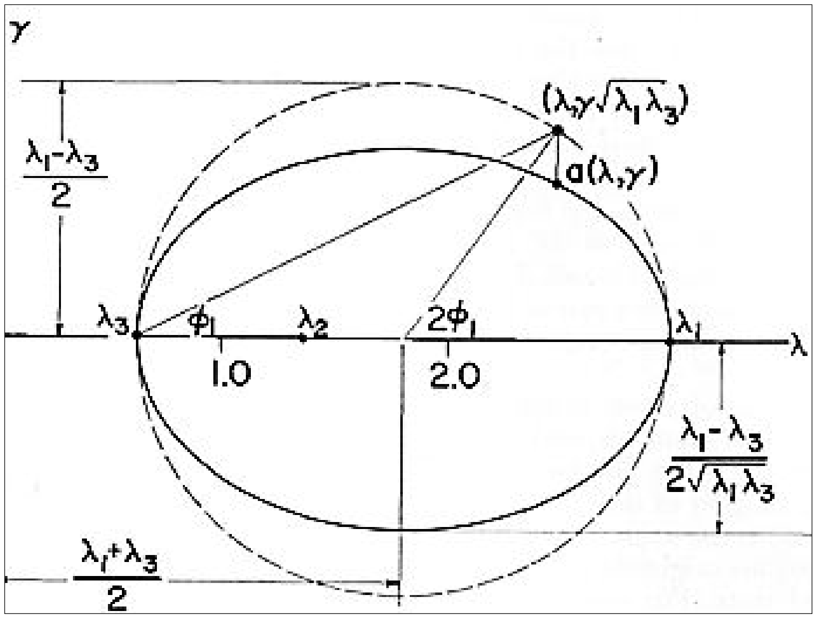

Figure 2 . Mohr diagram for the unstrained state. For a plane strain defined by λ_1 and λ_3 the elongation and unit shear of all lines are on the ellipse. An arbitrary line, a, at an angle $\emptyset_1$ to the direction of λ_1 has strain components $a(\lambda,\gamma)$.The point $a(\lambda,\gamma)$ *is* found by dropping vertically from the point where a line of inclination *2* $\emptyset_1$ intersects the construction circle (dotted). The construction circle is drawn through λ_1 and λ_3. The area change in the plane of strain is $\sqrt{\lambda_1\lambda_3}-1$.

$$\lambda' = \lambda'_1 a'^2_a + \lambda'_2 a'^2_2 + \lambda'_3 a'^2_3, \tag{5}$$

$$\gamma'^2 = (\lambda'_1-\lambda'_2)^2 a'^2_1 a'^2_2 + (\lambda'_2-\lambda'_3)^2 a'^2_2 a'^2_3 + (\lambda'_3-\lambda'_1)^2 a'^2_3 a'^2_1, \tag{6}$$

where λ' = 1/λ, $\gamma' = \gamma/\lambda$, $\lambda'_1 = 1/\lambda_1$, etc., and a'_1 , a'_2 , a'_3 are direction cosines defining a direction in the strained state.
Again, for strain in the $\lambda_1\lambda_3$ plane and for λ_2 equals 1,

$$\lambda' = \frac{\lambda'_1+\lambda'_3}{2} + \frac{\lambda'_1-\lambda'_3}{2}\cos 2\emptyset'_1, \tag{7}$$

$$\gamma' = -\frac{\lambda'_1-\lambda'_3}{2}\sin 2\emptyset'_1, \tag{8}$$

where the angle $\emptyset'_1$ now gives the direction in the strained state of a line whose strain is λ and γ (or equivalently, λ' and γ'). These are the equations of a circle in co-ordinates λ' and γ'.This is defined as the Mohr diagram for the strained state (Figure 4). All states of strain regardless of area change are represented by circles in this Mohr diagram; in the Mohr diagram for the unstrained state only states of strain with no area change are circles. The angle $\emptyset'_1$ is laid off differently from $\emptyset_1$ in the two diagrams (Compare Figures 2 and 4).

To emphasize the difference between the present use of the Mohr diagrams and the more familiar usage for stress and infinitesimal strain, typical examples are shown in Figure 5. Usage of the two Mohr diagrams for finite strain is shown for a two-dimensional pure strain in Figure 6. The position of an arbitrary line is shown in the unstrained state at a and in strained state of a'.The strain for this direction is shown in the two Mohr diagrams.

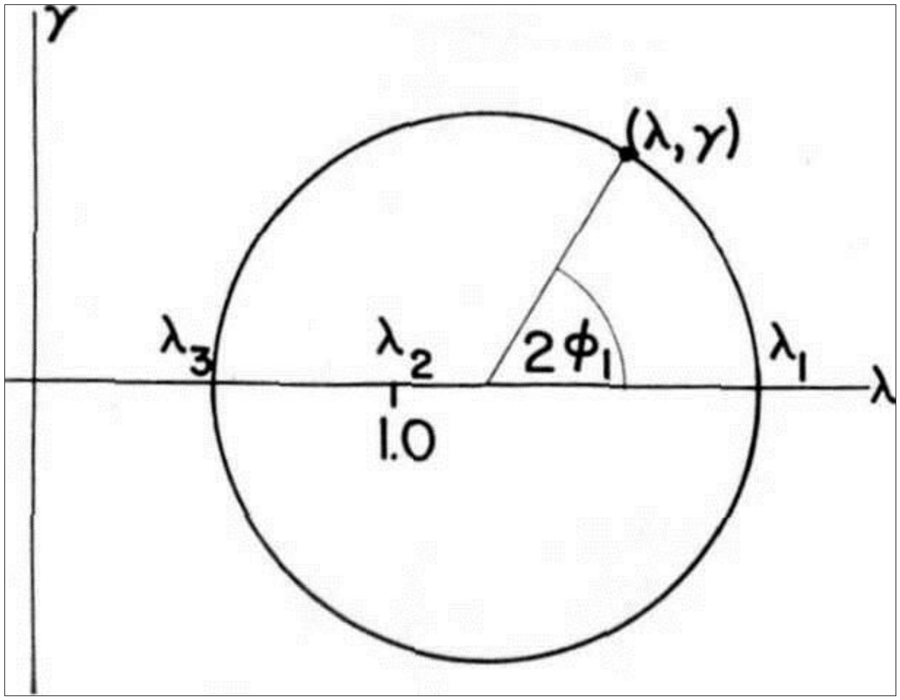

Figure 3. Mohr diagram for the unstrained state for constant area. The ellipse of Figure 2 becomes a circle, and λ_1 equals $1/\lambda_3$.

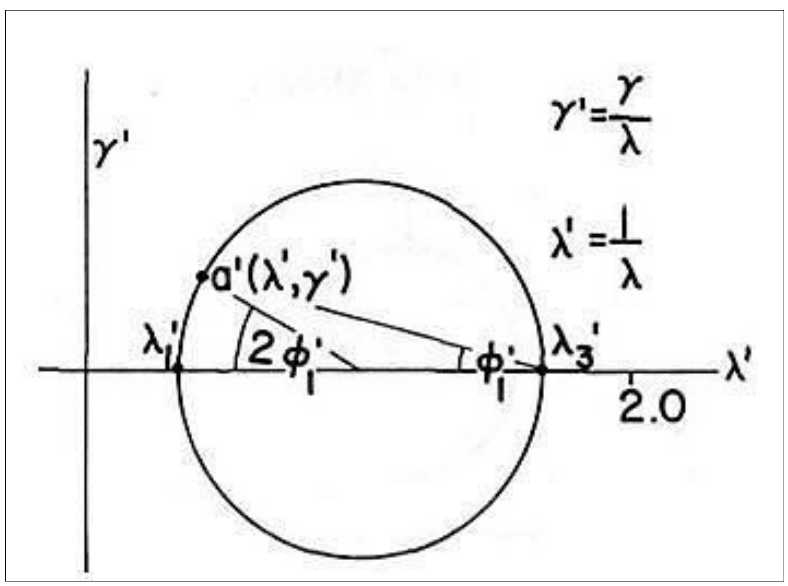

Figure 4 . Mohr diagram for the strained state.The principal quadratic elongations are the same as for Figure 2. In this diagram all strain states regardless of area change plot as circles. An arbitrary line in the strained state, a', which is at an angle $\emptyset'_1$ to the direction of λ_1 (or to λ_1 as they are parallel for pure strain), has components $a'(\lambda',\gamma')$ which are found on the circle through λ'_1 and λ'_3.in strained state of a'.The strain for this direction is shown in the two Mohr diagrams.

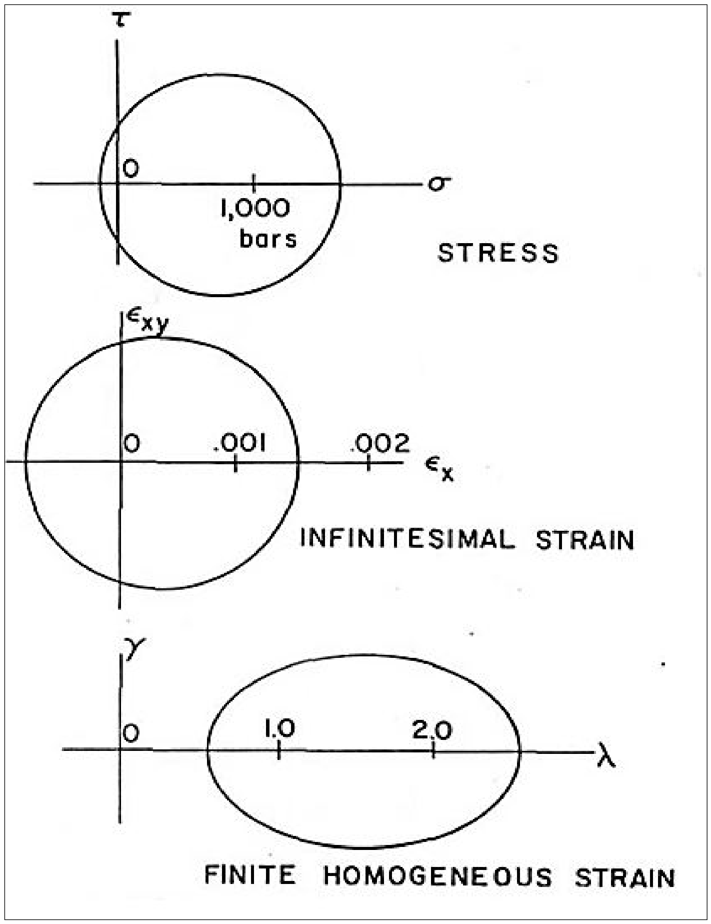

Figure 5. Mohr representations. The Mohr diagram for the unstrained state for finite homogeneous strain (bottom) is compared with the more familiar Mohr circle for stress (top) and for infinitesimal strain (middle). Note that for the finite-strain diagram the ellipse must lie to the right of the origin, because of the definition of λ.

Many of the properties of strain are easily visualized in the Mohr diagrams:

1. Only two lines ($\emptyset'_1$, $\emptyset_1 = 0°, 90°$) do not change direction as a result of strain. These are the intercepts of the Mohr ellipse or circle with the λ axis or the λ' axis. These lines undergo greatest and least strains, and for these directions γ equals zero. Only pure strain is considered here, and this by definition means that the directions of principal strain do not rotate, with respect to co-ordinates beyond the regions of strain.

2. All lines other than those coinciding with the directions of principal strain rotate with respect to one another. If the orientation of a line in the unstrained state is defined by $\emptyset_1$ then its position after strain will be the angle $\emptyset'_1$ may be found from the Mohr diagram for the strained state if the components of strain λ and γ, for the line are known and the principal strains are known. The quantities λ' and γ' are computed for the line, and its position on the circle in the $\lambda'\gamma'$ diagram found. This gives $\emptyset'_1$ and hence $(\emptyset'_1 - \emptyset_1)$, which is the rotation of the line as a result of strain. An alternate method of determining the change of direction which a line undergoes during strain is as follows. From Figures 2 and 4, for the line whose strain components are λ and γ.

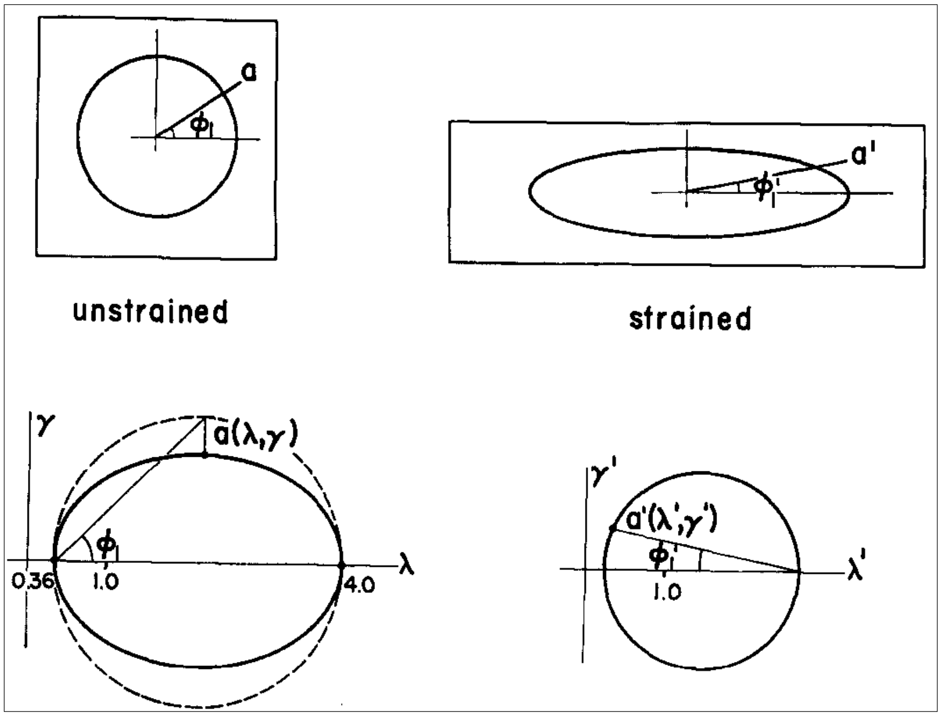

Figure 6 . Mohr diagrams for strain. A square and a circle are shown before and after a homogeneous pure plane strain denned by λ_1= 4.0 and λ_3= 0.36. The circle (upper left) transforms into an ellipse (upper right). If the circle has unit radius the ellipse is a section of the strain ellipsoid in the principal plane containing λ_1 and λ_3. An arbitrary line, a, before strain, has the strained orientation a'. The strain components, ϒ and λ, of this line are shown below referred to the unstrained state (lower left) and to the strained state (lower right). Area change here is +20 per cent.

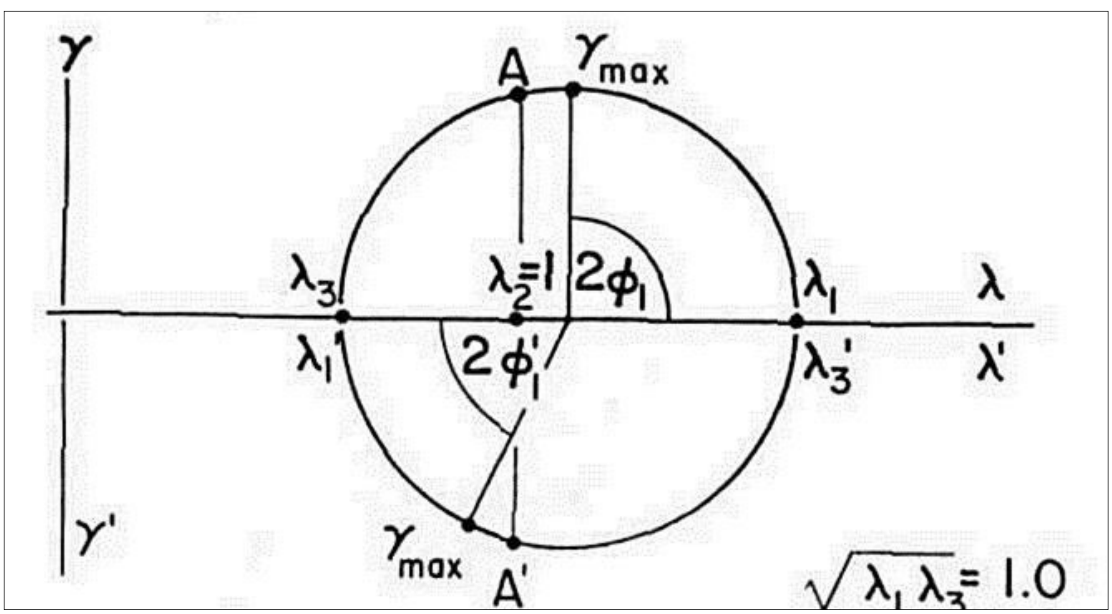

Figure 7 . Properties of strain in the Mohr diagrams. The Mohr diagram for the unstrained state is plotted above the abscissa, that for the strained state below. The position of the line of γ_{MAX} before and after strain is shown; $\emptyset'_1$ is always less than $\emptyset_1$for this and all lines. For this strain at equal area the planes of circular section and lines of no distortion coincide at the point A' in the strained and at A in the unstrained state.

$\tan \emptyset_1 = \frac{\gamma\sqrt{\lambda_1\lambda_3}}{\lambda - \lambda_3}$,

$\tan \emptyset'_1 = \frac{\gamma'}{\lambda'_3 - \lambda'} = \frac{\gamma/\lambda}{1/\lambda_3 - 1/\lambda} = \frac{\gamma\lambda_3}{\lambda - \lambda_3}$,

By combining these, we have the result

$$\tan \emptyset'_1 = \tan \emptyset_1 \sqrt{\frac{\lambda_3}{\lambda_1}}, \qquad (9)$$

which gives the new direction of a line in terms of the old direction and the principal strains? This result was derived by Wettstein (1886, p. 33) and used by Breddin (1956a, p. 264).

As an example, let $\emptyset_1$ of line *a* in Figure 6 equal 30°. Then
$\tan \emptyset'_1 = \tan 30° \sqrt{\frac{\lambda_3}{\lambda_1}} = 0.3 \tan 30°$,

$\emptyset'_1 = 10°$,

and the rotation of the line *a* during strain is $(30° - 10°)$ or 20° toward the λ_1 axis.

3. Two lines originally perpendicular in the unstrained state are represented by points on the end of a diameter in the Mohr diagram for the unstrained state. Any two such lines always undergo the same unit shear. The points corresponding to the two lines are not at the ends of a common diameter in the Mohr diagram for the strained state.

4. The maximum unit shear γ_{max}, is of magnitude

$$\frac{\lambda_1 - \lambda_3}{2\sqrt{\lambda_1\lambda_3}}$$

and is the unit shear of lines originally at $\pm 45°$ to the λ_1 and λ_3 axes of principal strain. Such lines always rotate in such a way as to approach the direction of maximum extension (Figure 7).

5. Planes of circular section would be represented in plane strain by lines whose quadratic elongation is the same as λ_2. Lines of unchanged length are lines whose quadratic elongation is unity. When λ_2 equals unity both of these pairs of lines coincide. (Figure 7)

6. Directions of maximum unit shear and directions of circular section or unchanged length never coincide when strain has taken place at constant volume. When volume has changed they may coincide. For example the direction of γ_{max} is a direction of unchanged length when the center of the Mohr ellipse is at (1, 0). This requires a large volume decrease.

7. For simple shear, which is equivolumnal plane strain, the motion is analogous to sliding of a deck of cards. A straight line inscribed on the edge of the cards remains a straight line. If the amount of slip of a unit thickness of cards is S then the Mohr diagram for the unstrained state, here a circle, goes through the two points $(1, \pm S)$ and intercepts the λ axis so that the quantity $\lambda_1\lambda_3$ equals unity. The lines of unchanged length in the unstrained state are ab and bc (Figure 8).

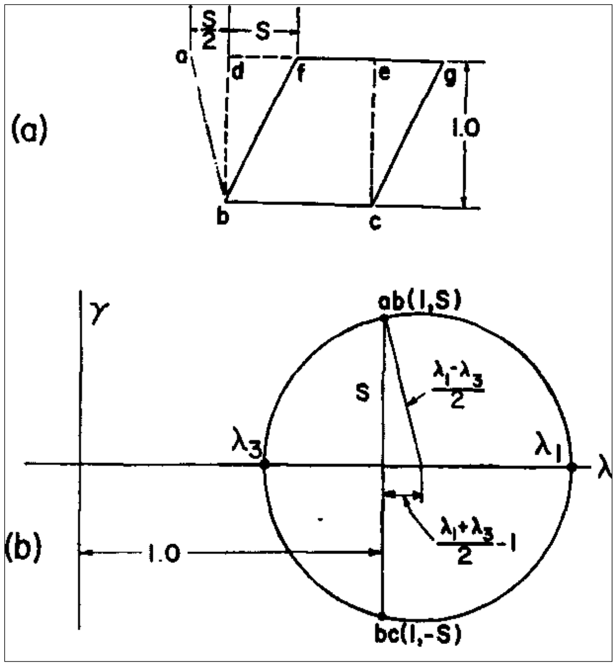

Figure 8 . Simple shear. The geometry of simple shear is shown in (a).Square $bdec$ is deformed into rhomb $bfgc$.Lines parallel with ab and bc are lines of unchanged length, and they have a unit shear of S. The Mohr diagram for the unstrained state is shown in (b) constructed from the two points $(1 \pm S)$ and by keeping λ_1 equal to $1/\lambda_3$as the area is constant in simple shear.

Any simple shear is completely defined by the quantity S which is the unit shear of the lines of unchanged length. The quantity S can be expressed in terms of the principal strains. From Figure 8,

$$S^2 = \left(\frac{\lambda_1 - \lambda_3}{2}\right)^2 - \left(\frac{\lambda_1 + \lambda_3}{2} - 1\right)^2 = \lambda_1 + \lambda_3 - \lambda_1\lambda_3 - 1,$$

Since$\lambda_1\lambda_3 = 1$,

$$S^2 = \lambda_1 - 2\lambda_1\lambda_3 + \lambda_3,$$

$$= \lambda_1 - 2\sqrt{\lambda_1\lambda_3} + \lambda_3,$$

From this

$$S = (\lambda_1)^{\frac{1}{2}} - (\lambda_3)^{\frac{1}{2}},$$

$$= \left(\lambda_2{}^2\right)^{\frac{1}{4}} - \left(\lambda_3{}^2\right)^{\frac{1}{4}},$$

$$S = \left(\frac{\lambda_1}{\lambda_3}\right)^{\frac{1}{4}} - \left(\frac{\lambda_3}{\lambda_1}\right)^{\frac{1}{4}},$$

Mohr diagrams for three-dimensional strain

In a general three-dimensional strain all three principal quadratic elongations are different from unity. The equations for the strain components are given above (equations $1, 2, 5, 6$), and Nadai showed (1950, p. 127-129) that they can be graphically represented in a somewhat more complicated form of the two-dimensional Mohr diagram discussed above. Figure 9 shows such a diagram for the unstrained state for a state of strain in which volume is not constant. It consists of

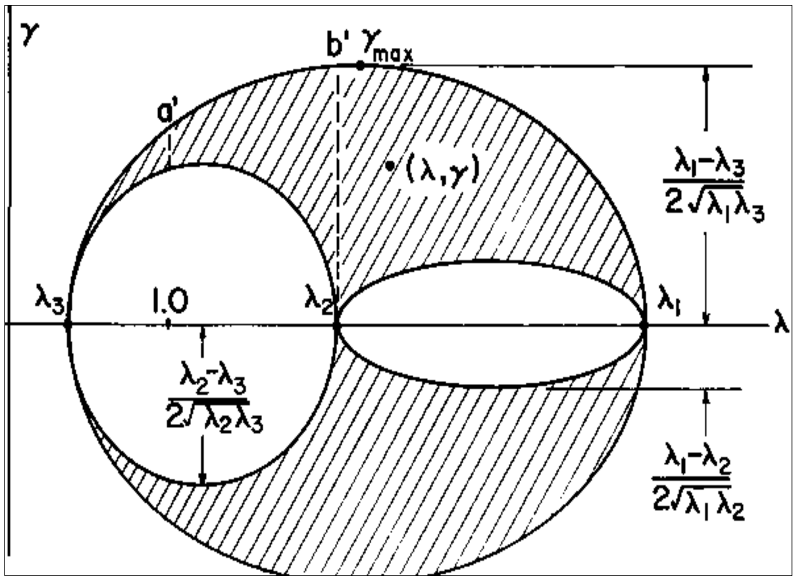

Figure 9 . Mohr diagram for three-dimensional strain. Typical Mohr diagram for the unstrained state for strain in which all three principal quadratic elongations differ from unity.The strain components of all lines lie within the area bounded by the three ellipses,shown cross-hatched.The Mohr diagram for the strained state consists of three circle for all states of strain.

three ellipses, which intersect the $\lambda - axis$ at the three principal quadratic elongations. Just as strain components of any line could be found for plane strain on the single ellipse, here they are contained in the area bounded by three ellipses (Figure 9). The strain in any arbitrary direction can be found by a method analogous to that for the three-dimensional Mohr diagram for stress and is given by Nadai (1950) and by Jaeger (1956, p. 37). For the strained state the Mohr diagram consists of three circles which intersect the $\lambda' axis$ at the three reciprocal principal strains.

To continue with the properties of strain, certain important features in three dimensions may now be added:

8. The point representing maximum unit shear (Figure 9) occurs on the ellipse joining λ_3 and λ_1 as this ellipse gives the strain components of lines in the $\lambda_1\ \lambda_3\ plane$, the maximum unit shear occurs in this plane.

9. The strain in any of the *principal planes,* which are planes containing two principal quadratic elongations, is independent of the value of strain perpendicular to that principal plane.

10. Equivolumnal strain does not necessarily result in ellipses becoming circles in the diagram for the unstrained state, as was the case for plane strain. For certain strains these ellipses do become circles, but only one at a time.

11. Lines of unchanged length are located along the dotted line a' in Figure 9, and all lines lying in the circular sections are located along the dotted line b'.

Geological misconception of strain

Three points of confusion frequently encountered in geologic literature from about 1900 to the present pertain to (1) the orientation of special directions such as the direction of γ_{MAX} in terms of the strain ellipsoid, (2) a supposed connection between large strain and elasticity, and (3) a supposed connection between homogeneous strain and properties of the strained material.

The first point is discussed under properties 4, 5, 6 and 11 above and should be easily visualized for any range of principal strains and volume change by using the Mohr diagrams.Many authors regard finite homogeneous strain as restricted to elastic strain; some even restrict discussion of the strain ellipsoid and its properties to infinitesimal strain, or to the range of validity of Hooke's law. Elasticity is an assumed relationship between stress and strain. Strain is a purely geometric description of deformed material. Strain of any magnitude can be discussed without recourse to a stress-strain law. The third point is due to confusion of the mathematical and physical usage of the term homogeneous. Homogeneous strain as defined above may occur in isotropic or anisotropic, homogeneous or inhomogeneous, material. It is only necessary to consider a twinned calcite crystal to demonstrate that homogeneous strain can occur in anisotropic material. Calcite is highly anisotropic with respect to strength. As another example, homogeneous shear strain may be imposed on a book which has pages made of different materials. The book is therefore inhomogeneous in the sense that it is not physically identical at all points, although its strain is homogeneous, when seen on a scale which is large compared with the page thickness.

Summary

Of various strain components available, the quadratic elongation and unit shear of Nadai, appear best suited to the needs of and types of information available in geology. One of their great advantages is that the equations of finite strain and the properties of strain can be presented graphically. This graphical representation is akin to the Mohr circle for stress but involves here two such diagrams, one for the strained and another for the unstrained state. Use of equations of finite homogeneous strain does not require that material be isotropic with respect to strength, that it be homogeneous, or that the deformation be within the range of validity of Hooke's law.

Analysis of strain in geology
General statement

The strain components of an arbitrary line have been found above from known principal strains. The more relevant geologic problem is that of finding the principal strains given the strain of certain lines or circular objects. Geologic features from which measurements of strain can be made are conveniently divided into two classes on the basis of original shape: circular (including spherical and cylindrical) and noncircular. The method of strain analysis is examined for these two classes below with a view particularly to finding the minimum information needed and the minimum number of assumptions required for complete determination of strain axes.

Strain from circular features

Pebbles, oöids, and crinoid columnals have been commonly used because of their approximately spherical or cylindrical shape. Ernst Cloos (1947), Oftedahl (1948), and Breddin (1956 a) discuss in great detail the derivation of strain axes from them, and this is only summarized here.

Objects originally spherical deform into ellipsoids. The principal strains or strain ratios, depending upon whether the original radius is known, are found at once from the ellipsoidal axes. Principal strains can be found from strain ratios if volume change is assumed (Brace, 1960, p. 265). One such spherical feature is sufficient to fix strain axes.

The deformation of circular cylindrical features into elliptical cylinders gives at once from the two axes of the ellipse a pair of local principal axes for planes perpendicular to the axis of the elliptical cylinder. Local principal axes for a plane are the axes of the ellipse that is the intersection of the plane and the strain ellipsoid; they are the maximum and minimum quadratic elongation in the plane in question. When this plane is a principal plane, the local principal axes become the principal axes.

If local principal strains can be found for any three orthogonal planes (this pertains also to strain measured in noncircular features) the principal strains in three dimensions can be found analytically by methods (Nye, 1957, p.41,165) beyond the scope of this paper. They involve solution of the cubic secular equation formed from coefficients of X' and Y' for the three orthogonal planes. This may be visualized as the construction of a complete three-dimensional Mohr diagram such as Figure 9 from six points which would lie within the cross-hatched area.

Strain from noncircular features

Noncircular features suitable for strain measurement include primarily *line elements,* or lineation of various kinds. Generally two or more line elements of known original relative orientation are required. For example, two originally perpendicular lines in a brachiopod are the hinge line and the trace of the plane of symmetry on the brachial valve.

A large group of noncircular features which has received little attention is the primary sedimentary structures. Vertical sections through cross-bedding, ripple marks, and similar structures in which an original angle of repose might be estimated to within $5° - 10°$ could be used as well as originally vertical features such as animal burrows, Scolithus, columnar and co-noidal stromatolites, and mud cracks. In the bedding surface useful line elements are the crest of ripple marks, convolute bedding, and scour-and-fill structures, which are originally perpendicular to load-cast lineation (Crowell, 1955, p. 1357). Example 2 under Examples of Strain Analysis, below, illustrates the use of particular noncircular features such as these.

Before turning to specific examples, measurement of strain in noncircular features is explored in a general way. For simplicity we first examine features which lie in a principal plane of strain (so-called two-dimensional strain) and then features having an arbitrary orientation with respect to the strain axes (so-called three-dimensional strain).

Two-dimentional strain

It is not at first obvious even for two-dimensional strain just how much information is needed to determine the magnitude and direction of the two principal strains. This can be explored systematically by noting how far toward a complete solution a single line, a line pair, and various combinations of such data will go.A single linear element of known original length would yield a single value of λ for its direction. However, even if two are available, principal axes cannot be found. Three, however, determine the magnitude and direction of two strain axes

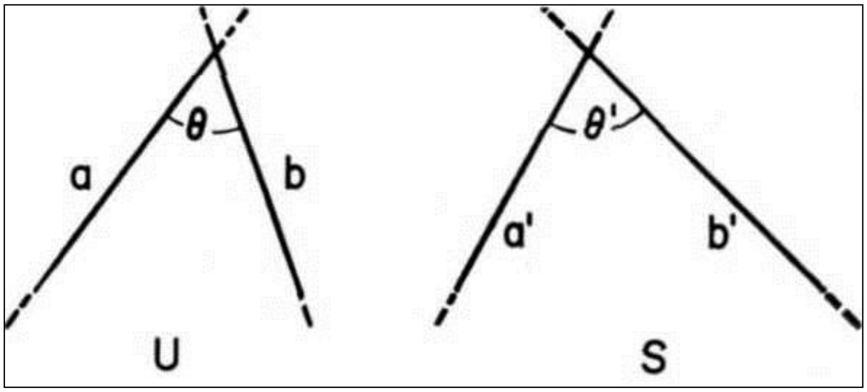

Figure 10 . Two directions at a known original angle. If θ is 90°, unit shear can be determined for this line pair. Otherwise no strain component can be measured. The directions are shown a; dotted lines at the ends, for they are simply directions, not line segments of known length. U is unstrained and S is strained state.

Because of the definition of γ two linear elements at an originally oblique angle (Figure 10) are not very useful. Nothing can be measured. If they were originally perpendicular y can be measured.

Three lines for which the original angular relationship is known (Figure 11) are more useful. To find a value of γ it is necessary to locate a right angle and then note how it has been changed. The lines ON and MN form such a right angle. Line ON has been constructed perpendicular to MP. In finite homogeneous strain the quadratic elongation of all lines in the same direction is the same; in the strained state the point N' can be located by using this property.

$$\frac{N'P'}{M'P'} = \frac{NP}{MP'}$$

Thus, In addition to γ the ratio of lengths ON and MN in strained and unstrained states is known. Calling ON and MN a and b respectively, then

$$\frac{\lambda_{ON}}{\lambda_{MN}} = \frac{\lambda_a}{\lambda_b} = \left(\frac{O'N'}{ON}\frac{MN}{M'N'}\right)^2 = \left(\frac{O'N'}{M'N'}\frac{MN}{ON}\right)^2,$$

Thus, starting with three lines of known original relative orientation, we determine unit shear and the ratio of quadratic elongations for an originally perpendicular line pair constructed from the three given lines. The same result might have been obtained had we started with an originally perpendicular line pair of known original length ratio.

The information derived so far is the unit shear and ratio of quadratic elongation of initially perpendicular lines, a and b. If, instead of length ratio, original lengths are known, then the Principal strains can be found directly (Brace, 1960, p. 266). Otherwise only the ratio of principal strains can be found, by the following method.

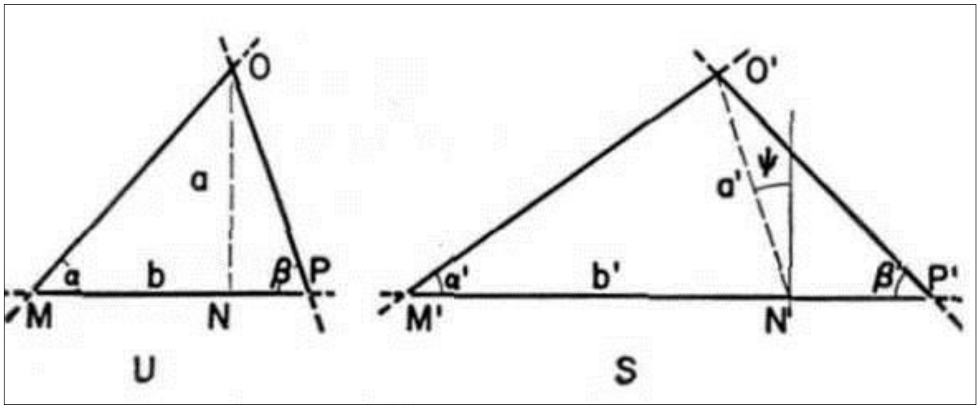

Figure 11 . Three directions at known original angles. Three directions are OM, OP and MP; their orientation in the unstrained state is shown at U and in the strained state at S. The line ON is constructed so that it is perpendicular to MN, ON and MN are two directions for which γ and the ratio of quadratic elongations, λ_a/λ_b, can be measured.

First a useful relationship is obtained. The quantity γ' for any line a is, from equation (8),

$$\gamma_a' = -\frac{\lambda_1' - \lambda_3'}{2}\sin 2\emptyset_{a1'},$$

and for line b

$$\gamma_b' = -\frac{\lambda_1' - \lambda_3'}{2}\sin 2\emptyset_{b1}',$$

where λ_1' and λ_3' are the reciprocal principal strains in the plane containing a and b, and $\emptyset_{a1}'$ and $\emptyset_{b1}'$ are the angles in the strained state between the direction of λ_1' (or equivalently, the direction of λ_1) and a and b, respectively. Dividing these we have

$$\frac{\gamma_a'}{\gamma_b'} = \frac{\sin 2\emptyset_{a1}'}{\sin 2\emptyset_{b1}'} = \frac{\gamma/\lambda_a}{\gamma/\lambda_b} = \frac{\lambda_b}{\lambda_a}.$$

The angle between aand b in the strained state is known. Call this angle C, where

$$C = \emptyset_{a1}' + \emptyset_{b1}',$$

Positive $\emptyset_{a1}'$ is the acute angle measured from the direction of λ_1' toward a, and positive $\emptyset_{b1}'$ is the acute angle measured from the direction of λ_1' toward b.
We can write

$$\sin 2\emptyset_{a1}' = \frac{\lambda_b}{\lambda_a}\sin 2\emptyset_{b1}' = \frac{\lambda_b}{\lambda_a}\sin(2C - 2\emptyset_{a1}'),$$

Solving for $\emptyset_{a1}'$ we obtain

$$\emptyset'_{a1} = 1/2\tan^{-1}\left(\frac{\sin 2C}{\lambda_a/\lambda_b+\cos 2C}\right),$$

$$\emptyset'_{b1} = 1/2\tan^{-1}\left(\frac{\sin 2C}{\lambda_b/\lambda_a+\cos 2C}\right), \qquad (11)$$

where

$$C = \emptyset'_{a1} + \emptyset'_{b1},$$

Knowing λ_a/λ_b from the preceding proposition and C, we obtained $\emptyset_{a1}{}'$ and $\emptyset_{b1}{}'$.In other words the orientation of the principal strains can be determined.The initial and final orientation of lines a and b are related according to equation (9) by

$$\tan\emptyset_{a1}{}' = \tan\emptyset_{a1}\sqrt{\frac{\lambda_3}{\lambda_1}},$$

and

$$\tan\emptyset_{b1}{}' = \tan\emptyset_{b1}\sqrt{\frac{\lambda_3}{\lambda_1}},$$

where $\emptyset_{b1}$ and $\emptyset_{b1}$ are the angles in the unstrained state between the λ_1 axis and lines a and b ,respectively. From the above equations,

$$\tan\emptyset_{a1}{}'\tan\emptyset_{b1}{}' = \tan\emptyset_{a1}\tan\emptyset_{b1}\frac{\lambda_3}{\lambda_1},$$

But, since lines a and b were originally perpendicular,

$$\tan\emptyset_{a1} = 1/\tan\emptyset_{b1},,$$

$$\tan\emptyset_{a1}\tan\emptyset_{b1} = 1,$$

Hence,

$$\tan\emptyset_{a1}{}'\tan\emptyset_{b1}{}' = \frac{\lambda_3}{\lambda_1}, \qquad (12)$$

The ratio of principal strains, λ_1/λ_3, is obtained from the values previously obtained for $\emptyset_{a1}{}'$ and $\emptyset_{b1}{}'$. Thus, from three line elements of known original relative orientation, the directions and ratio of two principal strains can be found.

The ratio of principal strains can also be found from an originally perpendicular line pair if geologic evidence independent of the strained feature indicates the direction of one of the strain axes. For example, the axis of flexural folds has been taken as the intermediate strain axis; in many cases a prominent lineation in the rocks is assumed to be parallel to a strain axis. Discussion of the validity of these assumptions is beyond the scope of this paper; each situation must be judged on its own merits.

Suppose that such an assumption is justified where we have two originally perpendicular lines, a and b (Figure 12): A prominent lineation, *OO,* is taken to be parallel with a strain axis. From the distortion shown, OO is the direction of the greater principal strain, designated as λ_1. The angles $\emptyset_{a1}'$ and $\emptyset_{b1}'$ are measured, and the ratio of principal strains, λ_1/λ_3, is obtained from equation (12).

The magnitudes of the principal strains cannot be determined from the ratio λ_1/λ_3 regardless of how many strained line pairs of unknown length are available, even if the direction of one of the principal strains can be deduced independently.

All this information are summarized in Table.

1.**THREE DIMENTIONAL STRAIN** : Strain of non-circular features has been explored above for features which lie in principal planes of strain. The results shown in Table 1 hold regardless of the magnitude of the principal strain perpendicular to the plane of the feature. For features lying in random planes with respect to strain axes, we note an immediate difficulty: the unit shear, γ, can only be measured in principal planes; the simple meaning found for y in Appendix *2* is not valid in random planes. Therefore, strain analysis using γ can only be made in principal planes. Using the methods of analysis presented here three principal strains can be found only by solving two or more two-dimensional problems. For noncircular features these must be in planes recognizable beforehand as principal planes. This is done most simply by noting originally perpendicular directions which have remained perpendicular through deformation: these directions are principal strain axes. For layered rocks which have undergone flexural folding one strain axis is probably parallel with the fold axis. Hence, one principal plane will be approximately perpendicular to fold axes.

Two three-dimensional cases, then, can be treated by the methods given here:

1. Principal strain ratios are measurable in two or three principal planes. From this the ratiosλ_1/λ_2, λ_1/λ_3, and $\lambda_2/\lambda_3$are found.
2. Principal strains are measurable in two or three principal planes. Therefore, λ_1 , λ_2 , and λ_3 are found, and from these the volume change can be computed.

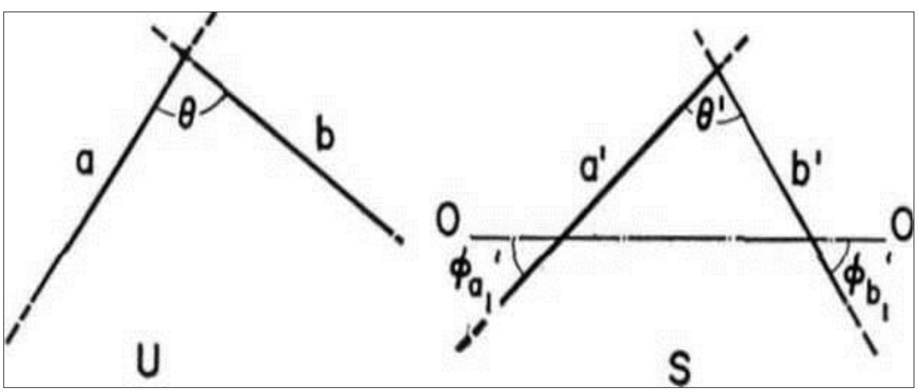

Figure 12 . Two directions and a strain axis. Two directions, a and b, whose original and final orientations with respect to one another are known and for which the orientation of a principal strain axis is known for the strained state.

Summary

Geologic features in which strain can be measured fall into two groups on the basis of original shape: circular (including spherical and cylindrical) and noncircular. The former include oöids, pebbles, vesicles, crinoid columnals, and clay balls. The latter include a large group of fossils and primary sedimentary structures in which the common feature is that original length or original relative orientation of linear elements is known.

The three principal strains of a general state of strain can be found by the methods given here (for other than originally spherical objects) only where strained features are available that lie in principal planes of the strain. The principal planes must therefore be recognizable beforehand. This is most easily done by noting originally perpendicular linear elements that have remained perpendicular during strain. For simply folded rocks the plane normal to the fold axis can probably be taken as one principal plane.

Various ways of finding two principal strains, or a principal strain ratio, are given for strained features that lie in principal planes. The most useful features are those with an originally circular shape. The most useful noncircular features are those of known original length; the next most useful are groups of three or more linear elements of known original relative orientation, and the least useful are perpendicular pairs of linear elements.

Examples of strain analysis

Two examples of two- and three-dimensional strain analysis are detailed below. Three other examples are given elsewhere (Brace, 1960, p. 265-267).

Example 1: A portion of a crystal has sheared in such a way (Figure 13a) that a line AB has changed direction to BC in passing from the unstrained region below the line BB' into the region of homogeneous strain above BB'. Volume change is zero. Determine the magnitudes and locations of the principal strain axes referred to both strained and unstrained regions. Assume that all shearing occurred in the plane of the sketch, and that BB' is of unchanged length.

This is a problem of simple shear. The quantity $S = \cot\theta_2 - \cot\theta_1$, *which equals 0.48*. This is all that is necessary to construct the Mohr diagrams for the strained and unstrained states, noting that BB' is a line of unchanged length and from the zero volume change that $\lambda_1\lambda_2\lambda_3 = 1$, as all the strain is in one plane, $\lambda_2 = 1$, and hence $\lambda_1\lambda_2 = 1$. The points (*1,0.48*) are laid off and the circle drawn so that $\lambda_1 = 1/\lambda_3$.The magnitudes of the principal strains can be read from the diagrams as $\lambda_1 = 1.6$ and $\lambda_3 = 0.62$.The orientation of principal through the angles $\emptyset_1$ and $\emptyset_1'$ which relate the direction of λ_1 to the direction of unchanged length BB'. Short lines having the direction of principal strains are shown in Figure 13c.This problem is not one of pure strain because principal strains have been rotated during deformation; in Figure 13c principal strains represented by the short line pairs do not have the same orientation in strained and unstrained states. The amount of rotation cannot be obtained from the Mohr diagram but follows here from the statement of the problem.Example 2: Beds of sandstone contain cross-bedding and worm tubes which have been sheared as a result of folding. Figure 14a shows the folded beds as seen in a joint face normal to fold axes. There is no distortion parallel with fold axes, and volume change is assumed to be zero. Find the strain axes.

Table 1.two-dimensional analysis using noncircular features

The number of Line Elements available is Given with the known original orientation, The strain components that can be measured, and The Principal Strains that can be derived. In the last column, $\emptyset'$ means that the Orientation of Principal Axes can be found, where three Line Elements are not originally perpendicular, A perpendicular line pair, A And B, may be constructed from them.

Number of line elements available	**Original orientation**	**Strain components measurable**	**Principal strains derived**
2	Oblique	Nothing	Nothing
2	Oblique,of known lengths	λ	Nothing
2	Perpendicular	γ	Nothing
2	Perpendicular, of known length ratio	$\gamma, \lambda_a/\lambda_b$	$\lambda_1/\lambda_3, \emptyset'$
2	Perpendicular, of known length ratio	$\gamma, \lambda_a, \lambda_b$	$\lambda_1/\lambda_3, \emptyset'$
2	Perpendicular, orientation of principal strain axis determined independently	$\gamma, \lambda_a/\lambda_b$	$\lambda_1/\lambda_3, \emptyset'$
3	At known oblique angle	$\gamma, \lambda_a/\lambda_b$	$\lambda_1/\lambda_3, \emptyset'$
More than 3	At known oblique angle	$\gamma, \lambda_a/\lambda_b$	$\lambda_1/\lambda_3, \emptyset'$
3	At unknown oblique angle,known lengths	….	$\lambda_1/\lambda_3, \emptyset'$

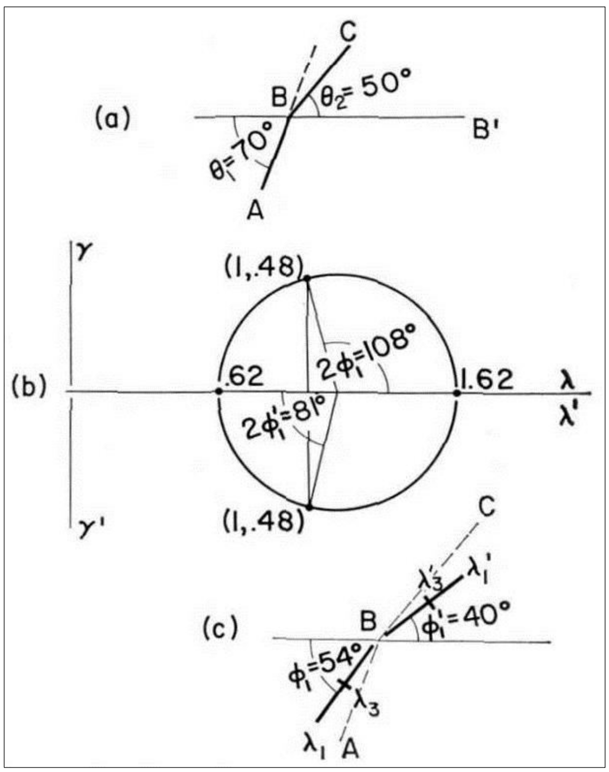

Figure 13 . Sheared crystal. A section parallel to the direction of slip is shown in (a) Material below the line BB' is unstrained, that above strained in such a way that the extension of the line AB moves to BC. The Mohr diagrams for this strain are constructed in (b). The orientation of principal strains deduced from the Mohr diagrams is shown in (c) by the heavy pairs of lines. Note that the directions of principal strains in strained (upper) and unstrained (lower) states in (c) are different, for this is not pure strain.

The angle of repose of the material in the cross-bedding is assumed to be 25°; the effect of an uncertainty of 5° is investigated below. As the worm tubes were normal to bedding, there are then three directions whose strained and unstrained relative orientations are known (Figure 14 c , b).
Using the method given in the previous section we find the strain components of the originally perpendicular line pair, the bedding, a, and worm tubes, b:

$\gamma = \tan 20° = 0.364,$

$\lambda_a/\lambda_b = \left(\frac{a'b}{ab'}\right)^2 = \left(\frac{a'b}{b'a}\right)^2,$

$= \left(\frac{\sin 52°}{\sin 58°} \tan 25°\right)^2 = 0.188,$

$C = 70°,$

Using equation(11),

$\emptyset_{a1}{}' = 1/2 \tan^{-1} \frac{\sin 140°}{0.188 + \cos 140°},$

$= 66°,$

$\emptyset'_{b1} = 70 - 66 = 4°,$

These results in equation (12) give

$\frac{\lambda_3}{\lambda_1} = \tan 4° \tan 66°,$

$= 0.16,$

The volume change was assumed to be zero; hence, $\lambda_1\lambda_2\lambda_3 = 1$ and the intermediate principal strain was assumed to be zero.
Hence,

$\lambda_1\lambda_3 = 1,$

From this,

$\lambda_1 = 2.5, \lambda_3 = 0.4,$

The principal strains are oriented as shown in Figure 14a, based on the angle $\emptyset_1{}'$ measured from the Mohr circle.
The effect of assuming that the angle of repose was 30° instead of 25° gives principal strains $\lambda_1 = 2.1$ and $\lambda_3 = 0.48$,
The angle $\emptyset_1{}'$ changes by about 2°. This difference in result is certainly within the anticipated accuracy of the methods discussed here.

Summary and conclusions

A method is developed for finding principal finite strains in deformed rocks from features which are of such a scale that they are included within regions of homogeneous strain. The question of whether this measured strain is total strain is not discussed, nor is the mapping of strain fields in particular structures. Finite homogeneous strain theory is applied to the analysis of strain with the help of a construction similar to the Mohr diagram for stress. Strain is purely a geometric property, and the use of the strain ellipsoid and its properties in strain analysis implies nothing about isotropy or homogeneity of the rocks involved or about the stress-strain relation during deformation.

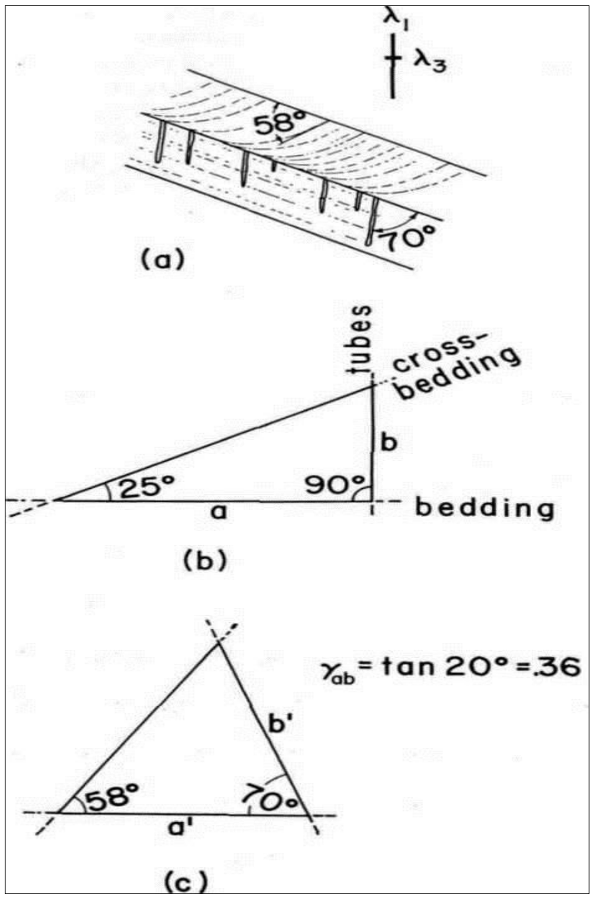

Figure 14. Deformed cross-lamination and worm tubes. Deformed cross-bedding and worm tubes (a), with bedding, give three directions of known original relative orientation (b). In the deformed state the three directions have the relative orientation shown in (c). The orientation of principal planes as determined in the text is shown by the heavy pair of lines in (a).

The immediate purpose of strain analysis in geology is determination of principal strains and volume change from individual measured components. The most suitable strain components appear to be the quadratic elongation, A, and unit shear, 7, as defined by Nadai (1950). Apart from being easily measurable and simply related to the general strain and the infinitesimal strain tensors, A and 7 can be used in a Mohr diagram. This enables various properties of finite strain to be easily visualized and also greatly simplifies certain geologic problems.

Finite strain can be described by means of two Mohr diagrams for the strained and for the unstrained states, which replace the single Mohr diagram used for infinitesimal strain. A general three-dimensional strain in many instances can be regarded as separate two-dimensional strains. Geological features from which strain can be measured are grouped according to original shape as circular (including spherical and cylindrical) and noncircular. The former include oöids, spherical pebbles, crinoid columnals, certain algal structures, vesicles, clay balls, and some concretions. Local principal strains arc found at once from the axes of ellipse or ellipsoid. The three principal strains or strain ratios are determinable, and for the former the volume change can be found. If volume change is assumed principal strains can be found from principal strain ratios.

Noncircular features include fossils (skeletons, bivalves, worm borrows, graptolites, and certain algal structures) and primary sedimentary structures (ripple mark, cross-lamination, mud cracks, groove-cast, load-cast, convolute bedding, and scour-and-fill), in which the common feature is two or more identifiable directions of known original relative orientation. In noncircular features λ and γ are usually measured, although less commonly λ or γ alone. Owing to the definition of γ it can only be measured if the noncircular feature is in a principal plane of the strain. This is inconvenient because it presupposes that principal planes can be recognized beforehand. This is done most simply by noting originally perpendicular linear elements which have remained perpendicular during strain.

The three principal strains of a general state of strain can be found by the methods given here only if strained features are present that lie in principal planes of the strain. This restriction does not apply to originally spherical features.

Several general conclusions regarding strain analysis in geology are:

1. In addition to oöids, spherical pebbles, and crinoid sections, a large group of fossil remains and impressions and primary sedimentary structures are available for strain measurement.
2. Use of the Mohr diagrams and strain components introduced by Nadai (1950) and illustrated here should solve the problem of visualization of the properties of finite homogeneous strain and make possible the easy solution of many geologic problems, some of which require trial and error (and hence are very tedious analytically).
3. The most useful features at present appear to be those originally spherical or disc shaped and those single linear features in which extension can be measured. Next most useful are groups of three linear elements of known original orientation, and least useful are originally perpendicular line elements.
4. Under ideal but rather rare conditions, volume change can be determined.
5. The minimum number of a priori assumptions needed to solve a strain problem depends on the specific problem and on the kinds of strained features available. Unless independent estimate of volume change is made the magnitudes of three principal strains probably will be obtained only in rare situations. In some instances it may be necessary to estimate the direction and magnitude of one of the strain axes independently of the strained feature. This can be done, with varying degrees of certainty, from the gross symmetry of the deformed rocks or from the character of fault movements but can probably be avoided, when the appropriate strained features are available, by use of methods of three-dimensional analysis beyond the scope of this study. In general no other assumptions about shape or orientation of the strain ellipsoid need to be made.
6. Accuracy of measurement of strain and volume change possible is probably in keeping with typical geologic measurement of attitude, or of fault displacement. Strain is a much more precise measure of the flow of rock than the usual visual estimate of thickening and thinning. Although strain measurement is a little-known technique in geology and requires subtle and imaginative use of minor structures combined with skill in manipulation of finite strain theory,

its study seems warranted. Strain is the most fundamental way of describing deformation, and from knowledge of strain in various geologic structures it should be possible to reconstruct their formation more accurately than is now possible**.**

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