EQUATION OF MOMENTUM AND CONTINUITY OF RIVER NETWORK

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Abstract: *This paper exhibits the relation between the structure of river networks and the features of their geomorphologic importance. The mathematical formulation and connectivity of a drainage network is the subject to relate contributing areas and the network geometry. In view of the connectivity conjecture, Horton's* *bifurcation ratio R_B tends, for high values of Strahler's order Ω of the basin, to the area ratio R_A, and Horton's length ratio R_L equals, in the limit, the single-order contributing area ratio R_a. The relevance of these arguments is examined by reference to data from real basins. Recent empirical results from the geomorphological literature (Melton's law. Hack's relation. Moon's conjecture) are viewed as a consequence of connectivity. It is found that in Hortonian networks the time evolution of contributing areas exhibits a multifractal behavior generated by a multiplicative process of parameter $1/R_B$. The application of the method of the most probable distribution in view of connectivity contributes fresh inroads toward a general formulation of the geomorphologic unit, in particular generalizing its width function formulation. A quantitative example of multifractal geomorphologic response of idealized networks based on the study of River Denwa of Pachmarhis, India (for which $R_B = R_A = 4. R_L = 2$).*

Introduction

The geomorphologic response of a drainage network to an impulsive net precipitation (the instantaneous geomorphologic unit) is viewed as the probable density function of travel times to the water particles penetrated uniformly in space *[Rodriguez-lturhe and Valdes,* 1979; *Gupta et al,* 1980; *Vissar, Heerdink, Broers, Bierkens*,2010;] Issues on the correct measures of channel networks are studied in this paper because of their implications on travel time evaluations.

Importance is given to measure the natural irregularity of channel networks. Irregular shapes are related to measurement through fractal geometry [*Mandelbrot*, 1983; *Ramkumar,* 2012;] which observes with scale symmetries are exact and statistical. The basic structure generates all details of the object at any scale of observation and that there exists a measure of the object scaling with a power law. In recent papers self-similarity in a river basin has been suggested both on theoretical and experimental grounds *[Tarboton et al*., 1988, 1990; *La Barbera and Rosso*, 1989, 1990; *Hjemfelt*, 1988; *Rosso et al*., 1991; *Evans, Dikau, Tokunaga, Omeri and Hirano,* 2003; *Vijay Gupta, Waymire,* 2010;]

Proof of fractal nature is the actual mainstream length L to the basin area $is\ A$ relation, which is taken as example in the form

$$L \propto A^{d'} \tag{1}$$

The empirical values of d' prominently vary from the Euclidean value of 0.5. The observations of *Mandelbrot's* [1983] regarding empirical and digital mapping shows that departures from Euclidean measures are the norm rather than the exception *[Lanfibein,* 1947; *Hack,* 1957; *Gray,* 1961; *Mesa and Gupta,* 1987; *Tarboton et al...* 1988, 1990; *Hjemfelt.* 1988; *Andre and Andre.* 1990; *Rosso et al.,* 1991;].

The very essential results of the fractal dimension of total length of a channel network has been accepted under the consideration that Horton's rules [e.g. *Smart.* 1972] applies exactly at any order of the river basin. Horton's empirical laws on bifurcation or length ratios are then considered as scaling laws. Two different regions of fractal scaling are seen in river networks, the former due to the sinuosity of individual rivers. [*Mandelbrot,* 1983; *Hjemfelt,* 1988; *Peter and Jorge, Ramirez,* 1998; *Zámolyi Székelya, Draganits, Timár* 2010;], and the latter due to the branching characteristics of the network *[Tar-botonetal.* 1988, 1990; *La Barbera and Rosso,* 1989, 1990; *Bruno and Foufoula-Georgiou, 2007; Taylor Perron, Paul Richardson, Ken Ferrier & Mathieu Lapotre, 2012;*], The fractal dimension D_Z of total length Z of Hortonian networks is suggested to be

$$D_Z = D_I \frac{\log R_B}{\log R_L} \tag{2}$$

where D_I is the fractal dimension due to sinuosity and R_B, R_L are Horton's ratios of bifurcation and length. It is concluded that the network itself may be viewed as space filling $(D_z = 2)$ and that the fractal dimension of individual rivers is $D_I \approx 1.14$ these results prove of interest to the issues dealt with in this paper.

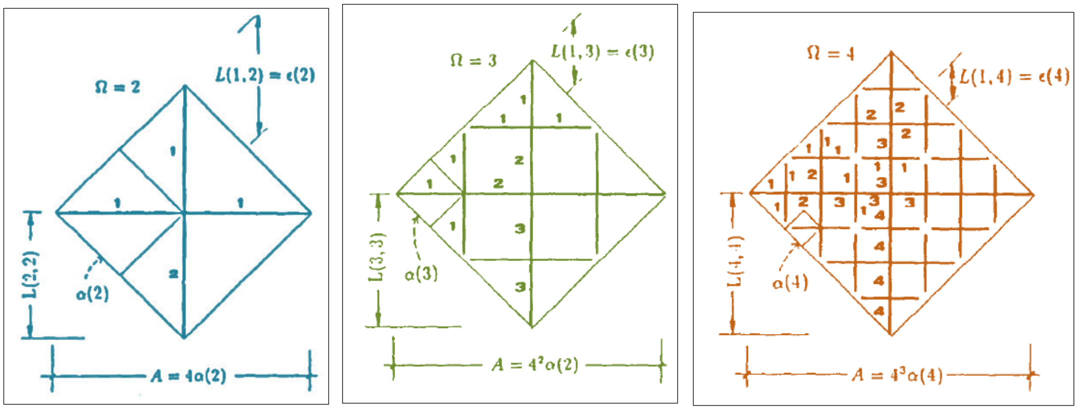

Figure. 1. The Denwa basin *[Dongre,1999,2012;]* . For every change of ruler (scale) every link generates four links, two resulting from the subdivision at half the length of the previous link, and two new links. In this basin $R_B = 4, R_L = 2, D_z = 2$.Strahler's ordering and details on the effects of a change of scale are shown with an example of sequence of length and area rulers $\varepsilon(\Omega), \alpha(\Omega)$. The basin of order 1 (the generator) is not drawn.

The River Network

The River networks and structure are usually been recognized as dendritics *[Shreve,* 1966; *Smart,* 1972; *Abrahams,* 1984; *Zaliapin, Foufoula-Georgiou, and Michael Ghil*,2010; *Thomas Neeson, Michael Wiley, Sara Adlerstein, Rick. Riolo,* 2012; *Dongre,* 2013;] Their structure is denned by Strahler's ordering [e.g., *Smart,* 1972;]. At each scale of observation drainage areas and the channel network are well defined, although a change in the scale modifies the resolution and new channels re-shape the drainage patterns. Nevertheless, persistence of symmetries of geomorphological quantities at different scales is observed *[Mandelbrot,* 1983; *Feder,* 1988; *Tarboton et al.,* 1988; *Sheridan Dodds and Daniel Rothman,* 2000;]. This fact suggested the application of concepts of the fractal geometry. The relation between the measure of any network set, $\wedge$, the unit of measure, ε, and the number of rulers, N_ε, needed to cover the set is

$$\wedge = N_\varepsilon \varepsilon^d \qquad (3)$$

When the set is self-affine $\wedge$ and d are called respectively fractal measure and fractal dimension *[Mandelbrot,* 1983; *Yadav, Bagla, Khandai,* 2010;]. If d is an integer the measure is Euclidean (a length, d = 1, an area, d = 2, a volume, d = 3, ⋯⋯).

The study conducted on the basis of structure and network of River Denwa (Figure 1), is convenient to select a sequence of rulers, $\varepsilon = \varepsilon(\Omega), \Omega$ =1, 2, ⋯⋯, which let the Strahler order, Ω, increase by one unit on changing $\varepsilon(\Omega - 1)$ to $\varepsilon(\Omega)$. A sequence of areal rulers, $a(\Omega)$, corresponding to the $\varepsilon(\Omega)(\Omega = 1,2,\dots,\infty)$ is chosen. The rulers are $\varepsilon(\Omega) = L(1,\Omega)$, the length of the shortest streams embedded in a network of order Ω, and $\alpha(\Omega)$ is defined as the drainage area of each link of length $L\,(1,\Omega)$. A relation between drainage areas and channel network may be derived through a connectivity conjecture which may be stated as follows: A point belongs to a drainage basin if there exists a channel connecting that point to the outlet of the basin.

To derive the implications of connectivity, the Denwa basin (Figure.1) is studied. The sequence of rulers, $\varepsilon, \alpha,$ is chosen so that the length ruler is the shortest link and the areal ruler is a square whose diagonal is the link. The following positions hold (Figure.1):

$$A = a(1) = R_a a(2) = \cdots = R_\alpha^{\Omega-1} \alpha(\Omega) \quad (4)$$

$$I_{\max} = \varepsilon(1) = R_\varepsilon \epsilon(2) = \cdots = R_\varepsilon^{\Omega-1} \varepsilon(\Omega) \quad (5)$$

where A is the drainage area; $R_a = R_A = a(i)/a(i-1)$; I_{max} is the maximum distance from source to outlet; and $R_\varepsilon = R_L = \varepsilon(i)/\varepsilon(i-1)$ are two suitable constant positive factors (later in this paper defined as Horton's ratios) and $L(1,\Omega) = L(I,1)/R_\alpha^{\Omega-1}$ is the average length of the shortest Strahler stream embedded in a network of order Ω.

The study reveals that the connectivity conjecture and structures of the number of area rulers does not increase more than the number of length rulers. In spite of the evidence that the form of such rulers is arbitrary (with the constraint of covering the set), the conjecture exactly states their number. Let N be such number; as in (3), connectivity implies

$$N_\varepsilon(\Omega) = N_a(\Omega) \quad (6)$$

Let A = A(Ω) and Z = Z(Ω) be the finite measures of the basin area and of the total basin length (i.e., scale independent); it follows that

$$Z = Z(\Omega) = N_\varepsilon(\Omega) \varepsilon(\Omega)^{D_z} \quad (7)$$

$$A = A(\Omega) = N_a(\Omega) \alpha(\Omega)^{D_A} \quad (8)$$

where $D_{Z,} D_A$ are the fractal dimensions of length and area. Applying then (6), (7), and (8) I obtain

$$\frac{Z}{A} \frac{\varepsilon(\Omega)^{D_Z}}{\alpha(\Omega)^{D_A}} = \rho \quad (9)$$

(i.e.,$A \propto Z$) where ρ is a finite constant called drainage density. The consequence of (9) is momentum which is dependent on ratios of drainage areas can be equally calculated by the ratio of the corresponding total basin lengths

Horton’s Law and River Networks

The river network analysis and scaling properties are explained by Horton's laws. Strahler's ordering [e.g., *Smart*, 1972; *Shreve*, 1966; *Pradhan, Ghose,* 2012;] yields (1) the bifurcation stream number law:

$$\frac{N(\omega-1,\Omega)}{N(\omega-\Omega)} \approx R_B \quad \omega = 2,\cdots,\Omega \quad (10)$$

and (2) the stream length law:

$$\frac{L(\omega,\Omega)}{L(\omega-1,\Omega)} \approx R_L \quad \omega = 2,\cdots,\Omega \quad (11)$$

where: $N(\omega,\Omega)$ is the number of streams in order w in a basin of order ω in a basin of order $\Omega(N(\Omega,\Omega) = 1)$; $L(\omega,\Omega)$ is the average length of streams of order ω; and R_B, R_L are Horton's numbers of bifurcation and length respectively. As a consequence one obtains

$$N(w,\Omega) = R_B^{\Omega-\omega} \tag{12}$$

$$L(\omega,\Omega) = \varepsilon(\Omega)R_L^{\omega-1} = L(\Omega,\Omega)R_L^{\omega-\Omega}$$

All natural basins have been found to satisfy the following limits and from connectivity (9) the following relationship is obtained:

$$1.5 \leq R_L \leq 3.5 \quad 3.0 \leq R_B \leq 5.0 \tag{13}$$

Let $\alpha(\omega,\Omega)$ be the average contributing area drained by a stream of order ω embedded in a network of order Ω referred to the areal ruler $\alpha(\Omega)$; $A(\omega,\Omega)$ be the average total contributing area drained by subbasins of order ω; $Z(\Omega,\Omega)$ be the total length of the network measured by the ruler $\varepsilon(\Omega)$; $N_\varepsilon(\omega,\Omega)$, $N_\alpha(\omega,\Omega)$be the number of rulers $\varepsilon, \alpha,$ respectively, needed to cover the area A and total length Z of the basin of order Ω up to its order ω.The Euclidean measures

$$A(\omega,\Omega) = N_a(\omega,\Omega)\alpha(\Omega) \tag{14}$$

$$Z(\omega,\Omega) = N_\varepsilon(\omega,\Omega)\varepsilon(\Omega)$$

often diverge and are generalized by the fractal measures which always converge:

$$A(\omega) = N_\alpha(\omega,\Omega)a(\Omega)^{D_A}$$

$$Z(\omega) = N_\varepsilon(\omega,\Omega)\varepsilon(\Omega)^{D_z} \tag{15}$$

On employing the relation

$$N_\alpha(\omega,\Omega) = \frac{1}{a(\Omega)}\sum_{i=1}^{\omega} N(i,\omega)a(i,\omega)$$

$$N_\varepsilon(\omega,\Omega) = \frac{1}{\varepsilon(\Omega)}\sum_{i=1}^{\omega} N(i,\omega)L(i,\omega) \tag{16}$$

It is possible to drive the following properties:

1. The total Euclidean basin length is *[Barbera and Rosso,* 1989; *Tarboton et al.,* 1990;]:

$$Z(\Omega,\Omega) = \varepsilon(\Omega)\frac{R_B^\Omega - R_L^\Omega}{R_B - R_L} = L(\Omega,\Omega)\frac{(R_B/R_L)^\Omega - 1}{(R_B - R_L) - 1} \tag{17}$$

2. The number of rulers, $\varepsilon(\Omega)$, needed to cover the network is

$$N_\varepsilon(\Omega,\Omega) = \frac{R_B^\Omega - R_L^\Omega}{R_B - R_L} \tag{18}$$

3. The fractal measure Z is [*Tarboton et al., 1988; Barbera and Rosso,* 1989;]

$$Z(\Omega) = \frac{R_L^{D_Z}}{R_B - R_L}L(\Omega,\Omega)^{D_Z} = \frac{R_B^{D_Z}}{R_B - R_L}L(\Omega,\Omega)^{D_Z} \tag{19}$$

with fractal dimension D_z given by (2). Also, in a Hortonian network $R_B > R_L$. Experimental data show (e.g., Table 2) that this condition is always satisfied.

4. From the definitions I have

$$A(\omega,\Omega) = \frac{\alpha(\Omega)}{\alpha(\Omega)} \sum_{i=l}^{\omega} N(i,\omega)\, a(i,\omega) \tag{20}$$

and from connectivity (9) the following relationship is obtained:

$$A(\Omega) = \frac{1}{\rho} \frac{R_L^{Dz}}{R_B - R_L} L(\Omega,\Omega)^{Dz} = \frac{1}{\rho} \frac{R_B^{Dt}}{R_B - R_L} L(\Omega,\Omega)^{Dz} \tag{21}$$

Where D_z is as in (2)

5. The connectivity and equation (9) the contributing area ratios [*Smart*, 1972;] are defined by

$$R_a(\omega,\Omega) = \frac{a(\omega,\Omega)}{a(\omega-1,\Omega)} = \frac{N_\alpha(\omega,\Omega)\,\alpha(\Omega)}{N_\alpha(\omega-1,\Omega)\,\alpha(\Omega)}$$

$$= \frac{N_\varepsilon(\omega,\Omega)\varepsilon(\Omega)}{N_\varepsilon(\omega-1,\Omega)\,\varepsilon(\Omega)} = \frac{L(\omega,\Omega)}{L(\omega-1,\Omega)} = R_L \tag{22}$$

$$R_A(\omega,\Omega) = A(\omega,\Omega)/A(\omega-1,\Omega)$$

$$= \frac{N(\omega-1,\Omega)}{N(\omega,\Omega)} \frac{\sum_{j=1}^{\omega} N(j,\Omega)a(j,\Omega)}{\sum_{j=1}^{\omega-1} N(j,\Omega)a(j,\Omega)}$$

Table 1. Experimental Constants of the Correlation Between the Basin Area and the main Stream Length

Reference	k_A	d'	$D_t(D_z = 2)$	$D_t(D_z = 1.8)$
Boyd[1978]	1.62	0.65	1.3	1.17
Gray[1961]	1.40	0.568	1.136	1.02
Leopal et al.[1964]	…..	0.6-0.7	1.2-1.4	1.08-1.26
Mueller[1973]	….	0.55	1.1	0.99
Langbein [1974]	0.9	0.56	1.12	1.01

(Data are taken from the literature [*Abrahams*, 1984; *Boyd*, 1978;].)

$$= R_B \frac{1-(R_L/R_B)^{\omega}}{1-(R_L/R_B)^{\omega-1}} \tag{23}$$

It is to be noted that (23) could be read as the following limit as $\omega \to \infty$:

$$R_A \approx R_B \tag{24}$$

This is asserted by *Rosso et al.* [1991, equation (15)] who found

$$R_B = R_A^{D_{l/2}} \tag{25}$$

for $R_A \geq R_B \geq R_{L}$.. This may be viewed as a result of the connectivity conjecture which implies space-filling networks ($D_Z = 2$)

6. Holding the connectivity conjecture, the Hack-Mandelbrot [*Feeler.* 1988; *Hjemfelt,* 1988; *Andre and Andre,* 1990;]

$$L(\Omega,\Omega) = k_{A}A^{d\prime} \qquad (26)$$

$$d\prime = \frac{1}{D_l}\frac{\log R_L}{\log\ R_B}$$

Inverting (21) with respect to the mainstream length I obtain

$$L(\Omega,\Omega) = \left[\rho\frac{R_B - R_L}{R_B^{D_i}}A\right]^{l/D_z} \qquad (27)$$

Comparing then Hack-Mandelbrot formulation with (27) I obtain the given result where $k_A \propto \rho$(Table 1).

7. In Hortonian basins, Melton's law [*Smart,* 1972;] relates empirically the ratio of Strahler's stream frequency, $S = \sum_{i=t}^{\Omega} N(i,\Omega)/A(\Omega)$,and the drainage density, $\rho = Z(\Omega,\Omega)/A(\Omega)$. Connectivity allows a theoretical definition of such a ratio in the case that the area A is a fractal measure (i.e., $A(\Omega)$) and the total length Z is a Euclidan measure($\text{i.e.}, Z(\Omega,\Omega)$). This is justified by the nature of the past empirical studies. It then follows that

$$\frac{S}{\rho} = \frac{R_B - R_L}{R_B - 1} \qquad (28)$$

The result follows froms (2) with $D_l \approx 1$, (19),(21) (recall that $\sum_{j=1}^{\Omega} N(i,\Omega) = \sum_{i=l}^{\Omega} R_B^{\Omega-1}$), the positions

$$\frac{S}{\rho^2} = \frac{A(\Omega)\sum_{i=l}^{\Omega} N(i,\Omega)}{Z(\Omega,\Omega)^2} \qquad (29)$$

$$\frac{S}{\rho^2} = \frac{1}{\rho}\frac{R_B - R_L}{R_B - 1}L(\Omega,\Omega)^{D_z - 2}\left(\frac{R_B}{R_L^2}\right)^{\Omega-1} \qquad (30)$$

Table.2 Horton's Parameters and Corresponding Computed Fractal Dimensions for Several River Networks of Pachmarhi

River	R_B	R_L	R_A	D_Z	D_A
Tawa	4.81	2.97	5.35	1.5	1.54
Denwa	3.69	2.61	4.05	1.36	1.45
Sonbhadra	3.11	2.07	2.80	1.56	1.44
Bainganga	3.76	2.63	4.35	1.37	1.52
Dudhi	3.13	1.82	3.29	1.90	1.98
Jambudip	4.1	2.18	4.71	1.81	1.99
Nagduwari	3.96	2.41	4.80	1.56	1.78
Bori	4.2	1.75	4.5	2.56	2.61
Gidh	2.7	2.0	5.1	1.43	2.35
Sawariya	3.9	2.3	4.5	1.6	1.8
Ganja Kunwar	4.1	2.3	4.6	1.69	1.83
Nagan	4.4	1.8	4.2	2.52	2.44
Gohara	4.9	2.5	4.6	1.73	1.67

and the requirement that Melton's ratio is a constant.

8. Experimental data in Tables 1 and 2 are taken from the geomorphological literature.from the data I infer a substantial agreement with the laws postulated by statement of connectivity.Apart from evalutions of Horton's parameters different from the expected values $D_Z \approx 2$, Corresponding values of R_A and R_B remain close.

9. Results may be compared with other experimental (or empirical) evidence:

First, Schumm's hypothesis *[Smart,* 1972;], which assumes $R_A(\omega, \Omega) \approx \text{const}$ $(\omega = 2,3, \ldots, \Omega)$, is in fact the result in (24) and the limits reported in the literature $(3.0 \leq R_A \leq 6.0$ [e.g., *Smart*, 1972;]) are close to the limits of RB, (equation (13)).

Second, experimental measures for $d', D_Z, D_A, D_l, R_A, R_{B'}$ and R_L, are collected in Tables 1 and 2. I note that if values of D_Z differ significantly from 2, large values of D_l are obtained seemingly in contrast with recent results *[Hjemfelt,* 1988; *Andre and Andre,* 1990;], defining a framework globally consistent with the connectivity conjecture and its consequences.

Third, Melton's relation for drainage areas in the context of topologically random river networks *[Smart,* 1972; *Ricardo, Brent Troutman, and Gupta* 2010;] allows an interesting comparison. From the analysis of 156 mature watersheds Melton proposed the relation:

$$S/\rho = 0.69 \quad (31)$$

At large values of Ω in the most probable topologically random network *[Shreve,* 1966;] I have $R_B = 4$ and $R_L = 2$ and therefore from (28) the ratio is

$$S/\rho = \frac{R_B - R_L}{R_B - 1} = 0.67 \quad (32)$$

which identifies very well with Melton's empirical value of 0.69 [e.g..*Smart,*1972;] Fourth, in

$$L(\Omega, \Omega) = R_L^{\Omega - 1} \varepsilon(\Omega) \quad (33)$$

$$L(\Omega, \Omega) = R_B^{(\Omega-1)/D_Z} \varepsilon(\Omega) = N(1, \Omega)^{1/D_Z} \varepsilon(\Omega) \quad (34)$$

Table 3. The Multiplicative Process Defined by the Contributing Areas at Various Distances From the Outlet, as a Function of Different Orders Ω in the Case of the Denwa Basin

Order Ω	Total Number of Links $Z(\Omega, \Omega)/\varepsilon(\Omega)$	Maximum Length $l_{max}/\varepsilon(\Omega)$	Width Function $N_{eff}(i, \Omega)$
1	4^0	2^0	$N_{eff}(1, 1) = 1$
2	4^1	2^1	$N_{eff}(1, 2) = 1$; $N_{eff}(2, 2) = 3$
3	4^2	2^2	$N_{eff}(1, 3) = 1$; $N_{eff}(2, 3) = 3$; $N_{eff}(3, 3) = 3$; $N_{eff}(4, 3) = 9$
4	4^3	2^8	$N_{eff}(1, 4) = 1$; $N_{eff}(2, 4) = 3$; $N_{eff}(3, 4) = 3$; $N_{eff}(4, 4) = 9$; $N_{eff}(5, 4) = 3$; $N_{eff}(6, 4) = 9$; $N_{eff}(7, 4) = 9$; $N_{eff}(8, 4) = 27$
5	4^4	2^4	$N_{eff}(1, 5) = 1$; $N_{eff}(2, 5) = 3$; $N_{eff}(3, 5) = 3$; $N_{eff}(4, 5) = 9$; $N_{eff}(5, 5) = 3$; $N_{eff}(6, 5) = 9$; $N_{eff}(7, 5) = 9$; $N_{eff}(8, 5) = 27$; $N_{eff}(9, 5) = 3$; $N_{eff}(10, 5) = 9$; $N_{eff}(11, 5) = 9$; $N_{eff}(12, 5) = 27$; $N_{eff}(13, 5) = 9$; $N_{eff}(14, 5) = 27$; $N_{eff}(15, 5) = 27$; $N_{eff}(16, 5) = 81$
Ω	$R_a^{\Omega-1}$	$R_\varepsilon^{\Omega-1}$	$N_{eff}(i, \Omega) = N_{eff}(i, \Omega - 1)$ $\quad i \leq R_\varepsilon^{\Omega-2} = l_{max}/\varepsilon(\Omega)$ $N_{eff}(i, \Omega) = (R_a - 1) N_{eff}(i - R_\varepsilon^{\Omega-1}, \Omega - 1)$ $\quad R_\varepsilon^{\Omega-2} < i < R_\varepsilon^{\Omega-1}$

The quantity $Z(\Omega,\Omega)/\varepsilon(\Omega)$ is the total number of links. The quantity $I_{\max^{I}}\varepsilon(\Omega)$ is the maximum total distance from source to outlet measured in $\varepsilon(\Omega)$units. $N_{eff}(i,\Omega)$is the number of links at distance $i\varepsilon(\Omega)$ from the outlet in a basin of order Ω.

This last relation resembles Moon's law *[Waymire,* 1989;] for which $L \propto \left(N(1,\Omega)\right)^{1/2}$ whenever $D_Z = 2,$ the most likely topological random network with fixed number of sources $N(1,\Omega)$being Hortonian with $R_B \approx 4$ and $R_L \approx 2$*[Shreve,* 1966].

It is then concluded that our connectivity conjecture yields insight into experimental (or empirical) evidence from the geomorphological literature

4. Denwa Curves and Path Probabilities of Hortonian Basins

From a hydrological point of view the description of the Hortonian basin is complete when all connected areas are identified and grouped in classes distinct by their distance l to the outlet. Choosing fractions l_i of the maximum distance $l_{\max}$from source to the outlet, $0 = l_0 < l_1 < \cdots < l_n < l_{max} = \varepsilon(\Omega)R_L^{\Omega-1}$, a partition is identified by subintervals of length $\varepsilon(\Omega)$, i, $e.$, $l_i = i\ \varepsilon(\Omega)$. The geometry of the connections defines a partition of the drainage area into subregions at the same distance from the outlet. On considering such a distance l_i, embedded in a network of order Ω, let $N_{eff}(i,\Omega)$be the number of paths available at such a distance, i.e., the width function *[Troutman and Karlinger,* 1984; *Gupta et al..* 1986; *Mesa and Gupta,* 1987; *Puente and Sivakumar,* 2003;], With reference to Denwa basin (Figure 1) the following relation is derived by induction (Table 3):

$$N_{eff}(l_i,\Omega) = N_{eff}(l_i,\Omega-1)\ i = 1,2,\dots,R_\varepsilon^{\Omega-2}$$

$$N_{eff}(l_i,\Omega) = (R_\alpha - 1)N_{eff}\left(I_i, -R_\varepsilon^{\Omega-2}(\Omega),\Omega-1\right) \tag{35}$$

$$i = R_\varepsilon^{\Omega-2} + 1,\dots,R_\varepsilon^{\Omega-1}$$

where $R_\varepsilon = \varepsilon(\Omega-1)/\varepsilon(\Omega) \approx R_L = 2$ as in (5); $R_\alpha = a(\Omega-1)/\alpha(\Omega) = R_A = 4.$ fact, at the scale $\Omega = 2, 1/4$ of the area is in the first half of the maximum distance, the remaining in the second. For $\Omega = 3,$ I have 1/16 in the first fourth, 1/4 ×3/4 in the second, 3/4 × 1/4 in the third, 9/16 in the fourth and so on. Varying Ω I obtain the characteristic multiplicative process of Table 3, and (35) by induction over Ω. The total (normalized) contributing areas and network lengths at distance l_i, from the outlet are defined by the following (equations (36) and (37)):

$$\mathbf{L}\left(I_{i,}\Omega\right) = \frac{1}{Z(\Omega)}\sum_{j=1}^{i} N_{eff}(j\varepsilon(\Omega),\Omega)\varepsilon(\Omega)^{D_{\mathfrak{L}}}$$

$$= \frac{1}{Z(\Omega)}\int_0^{l_i} N_{eff}(x,\Omega)\,dx \tag{36}$$

$$\mathbf{A}\left(I_{i,}\Omega\right) = \frac{1}{A(\Omega)}\sum_{j=1}^{i} N_{eff}(j\varepsilon(\Omega),\Omega)\alpha(\Omega)^{D_{\mathcal{A}}}$$

$$= \frac{1}{A(\Omega)}\int_0^{l_i} N_{eff}(x,\Omega)\,dA \tag{37}$$

where $D\mathbf{L}$,, $D_{\mathbf{A}}$ are suitable fractal dimensions. From the results of Table 3 and normalization I have

$$\mathbf{A}(x,\Omega) = \mathcal{A}(2x,\Omega-1)/R_\alpha\ 0 \le x \le 1/R_\varepsilon \tag{38}$$

$\mathbf{A}(x,\Omega) = \frac{1}{R_a} + \left(1 - \frac{1}{R_a}\right)\mathcal{A}(2x-1,\Omega-1)$

$1/R_\varepsilon \le x \le 1$

Where $x = i/R_\varepsilon^{\Omega-1}$

$\mathbf{A}\,(x,\Omega) = \sum_{j=1}^{R_i^{\Omega-1}x} N_{eff}(l_{j,}\Omega)/R_a^{\Omega-1}$

$\mathbf{A}(x,\Omega)$ is then the contributing area at distance x from the outlet.

From the relations (36) and (37) and from connectivity $(R_\varepsilon = R_L; R_\alpha = R_A = R_B)$ the following relations may be derived:

$$\mathbf{A}(l_i) = \mathbf{L}\,(I_i) \quad (39)$$

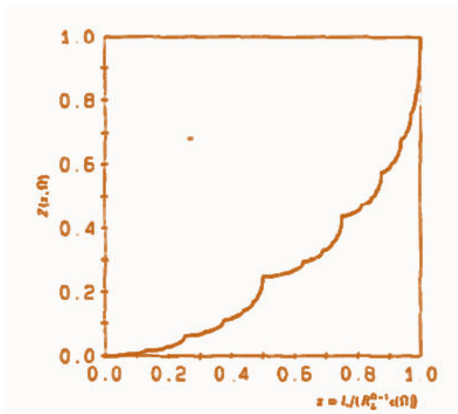

Figure. 2. Total length $\mathbf{L}(x)$measured from the outlet of Denwa network normalized by the total fractal length. Here $0 < x < 1$, the distance being normalized by the maximum distance from source to outlet $l_{\max}$A behavior similar to the devil's staircase is shown, i.e., the curve is everywhere continuous but has no derivative. Strahler's order for the computations is $\Omega = 11$.

$$\mathbf{A}\left(l_{i,}\Omega\right) = \frac{1}{R_B^{\Omega-1}}[(1/R_B)]^{\Omega-1-x^{(j)}} \quad (40)$$

$0 \le j \le \Omega$

where $X(j)$ is a suitable exponent $[Feder,$ 1988] to be computed recursively from Table 3. The term $1/R_B$ is the ruling factor of the multiplicative process.

The width function $N_{eff}(x)$has a multifractal structure. Figures 2 and 3 show the functions$\mathbf{L}(I_{i,})$, $N_{eff}(x)$ computed for the Denwa basin with $\Omega = 11$. The contributing length $\mathbf{L}(I_{i,})$has a behavior similar to that of the devil's staircase $[Feder,$ 1988;], I observe that the contributing length is always increasing even at no discontinuous points, as opposed to the staircase behavior. The proper staircase behavior holds for the sum of topological distances $N_{eff}(x,\Omega)$ only. Figure 4 shows the fractal dimensions $d(x)$ of $N_{eff}(x)$, obtained from the position:

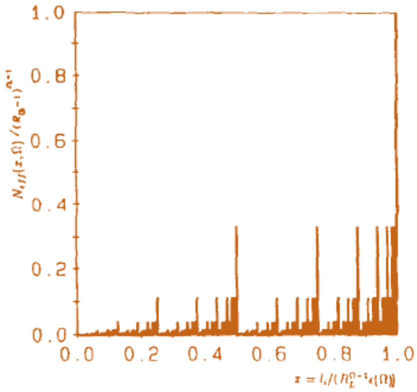

Figure. 3. The width function $N_{eff}(x, \Omega)$ as a function of the dimensionless distance from the outlet, defined by the number of active links at distance $0 \leq x \leq 1$. Strahler's order for the computations is $\Omega = 11$..

$$d(x) = \frac{\log N_{eff}(x)}{(\Omega - 1)\log R_L} \tag{41}$$

in the limit as $\Omega \to \infty$The minimum fractal dimension is 0 for $x = 0$ where one stream only is active, and the maximum is $\log(R_B - 1)/\log R_L = 1.5850$ because $(R_B - 1)^{\Omega-1}$ streams are effective at $x = l$.

To complete the description of the path structure, further information is required about the moments of $I_y = y\varepsilon(\Omega)$ weighted by the number $N_{eff}(I_y, \Omega)$ of active areas at the given distance:

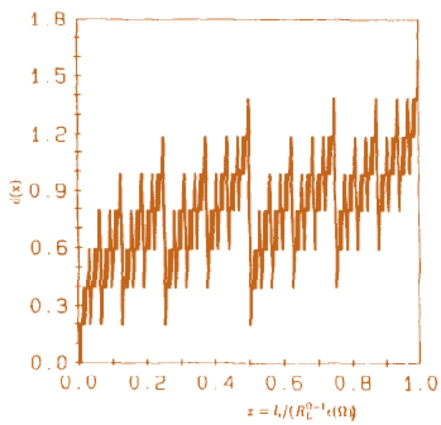

Fig. 4. Spectrum of multifractal dimensions $d(x)$of the width function $N_{eff}(x, \Omega)$. The spectrum is related to the Holder-Lipschitz exponent as in the work by *Feder* [1988]. Strahler's order for the computations is $\Omega = 11$

$$I(\beta) = \frac{\alpha(\Omega)}{A(\Omega, \Omega)} \sum_{\Upsilon=1}^{R_L^{\Omega-1}} N_{eff}(\Upsilon, \Omega) l_\Upsilon^\beta \tag{42}$$

Once connectivity is assumed $(R_A \approx R_B)$, in a self-similar Hortonian basin with $R_\varepsilon = R_L = 2$ each path drains the same area $\left(a(\Omega)/A(\Omega, \Omega) = 1/R_B^{\Omega-1}\right)$ and, upon introducing a common transformation *[Feder,* 1988; equation (6.19)], I obtain

$$I(\beta) = \frac{1}{R_B^{\Omega-1}} \sum_{\Upsilon=1}^{R_L^{\Omega-1}} \left(\frac{1}{R_B^{\Omega-1}}\right)^{\alpha HL(\gamma)} l_\gamma^\beta \tag{43}$$

where $\alpha_{HL}(\gamma)$ is Holder-Lipschitz' exponent of singularities. I observe that $\alpha_{HL}(\gamma)$ is related to the multifractal spectrum $d(x)$ of Figure 4 by the relation $\alpha_{HL}(x) = D_Z - d(x)$ Mean and variance of the distance l to the outlet may be computed analytically exploiting an iterative scheme [*Rinaldo et al.*, 1991; *Cheng, Dongfeng,* 2007; equations (21) and (22)] yielding the following values:

$$E[I] = I(1) = \frac{R_B - 1}{R_B}\frac{R_L}{R_L - 1}L(\Omega,\Omega) \quad (44)$$

$$\mathrm{Var}\,[I] = I(2) - I(1)^2 = \frac{(R_B-1)R_L^2}{R_B^2(R_L^2-1)}L^2(\Omega,\Omega) \quad (45)$$

In particular, I observe that every positive βth moment of the length distribution is finite owing to the fact that

$$\big(\alpha(\Omega)/A(\Omega,\Omega)\big)\sum_{\gamma=1}^{R_L^{\Omega-1}} N_{eff}(\gamma,\Omega) = 1$$

and that $I_\gamma^\beta \leq 1$.

5. Precipitation response

If the mass injected by an instantaneous pulse is thought of as constituted by N_T particles of equal mass, and N_i is the rate of arrivals at the outlet during the ith interval of a suitable partition of Δt of time $(i = 1,2,\ldots.)$, the arrival time distribution can generally be deduced from the multinomial expression for the joint probability [*Lienhard,* 1964, *Luis, Montiel, Bickel* 2012]:

$$P\big(N_l = n_{1,}\ldots, N_m = n_m\big) = N_{T!}\prod_{i=1}^{m}(g_i)^{n_i/n_{i!}} \quad (46)$$

where m is a cutoff number and g_i is the number of possible ways of placing n_i particles in the ith time partition, i.e., a probability proportional to the contributing area at time $t_i = i\Delta t$[*Gupta and Waymire,* 1983;]. Its time evolution with the elapsed time specifies the dynamics of contributing areas.

To know the expected arrival time distribution in a fractal channel network, the constraint of mass conservation is

$$\sum_{i=l}^{\infty} N_i = N_t \quad (47)$$

A matter of concern is related to the unclear definition of constraints of dynamic nature [*Lienhard,* 1964; *Lienhard and Meyer,* 1967; *Gupta and Waymire,* 1983; *Troutman and Karlinger,* 1985; *Rinaldo et al.,* 1989;]. The form of a dynamic constraint is

$$\sum_{i=l}^{\infty}(N_i/N_T)t_i^\beta = T^\beta > 0 \quad (48)$$

where β and $T$are positive constants. I recognize in (48) an expression related to the βth moment $I(\beta)$ in (43). In fractal networks β does not seem to have a unique value and, unlike previous investigations, I choose to drop the dynamic constraint in view of its arbitrary definition.

The optimal $\widetilde{N}_i$ are then obtained maximizing the logarithm of the probability in (46) accounting for the above constraints via Lagrange multiplication as in Boltzmann's or [*Lienhard and Meyer's* 1967;] analyses. The result is

$$\widetilde{N}_i = gi^{e^{-B_1}} \tag{49}$$

where B_I is a Lagrange multiplier. In the absence of dynamic constraints, in fact, all possible arrangements are equally likely. The travel time $f(t_i)$ is given by the following equation:

$$f(t_i) = \frac{\widetilde{N}_i}{\Delta t \sum_{i=1}^{\infty} \widetilde{N}_i} = \frac{g(t_i)}{\Delta t \sum_{i=1}^{\infty} g(t_i)} \tag{50}$$

The crux of the matter is therefore the definition of the time evolution of the number of distinct paths leading to the outlet at time t_i qualified by the function $g_i = g(t_i)$ the function $g_i$is proportional to the rate of contributing area at time t_i [*Gupta and Waymire,* 1983;]. In view of the limit equivalence (39) of ratios of contributing areas $\mathcal{A}(l,\Omega)$at distance I from the outlet and of contributing lengths $\mathfrak{T}(l,\Omega)$ I obtain

$$g_i = \mathbf{A}(l(t_i + \Delta t), \Omega) - \mathbf{A}(l(t_i), \Omega)$$

$$= \mathbf{L}(l(t_i + \Delta t), \Omega) - \mathfrak{T}(l(t_i), \Omega) \tag{51}$$

where $0 \leq l(t) \leq 1$ indicates that the contributing distances evolve with time as required by the dynamics of the process. From (36) I may write

$$\frac{g_i}{\sum_{i=1}^{\propto} g_i} = \frac{\int_{l(t_i)}^{l(t_i+\Delta t)} N_{eff}(x,\Omega) dx}{Z(\Omega)} \tag{52}$$

The correct meaning of (52) is as observed: A particle reaches the outlet at time t_i; the underlying equation of motion for the particle defines the trajectory $X_t(t; 0, I)$, a vectorial function; and $X_0 = l$is the initial position of the particle at $t = 0, \text{i.e.}, X_0(0; 0, I) = I$with probability one. The dynamic model considered is as follows:,

$$X_t(t; t_0, X_0) = X_0 + \int_0^t u\left(X_{\tau,}\tau\right) d\tau + X_B \tag{53}$$

where $u = v + u'$ is the general convection field X_B is a Brownian motion component. Here $v = \langle u \rangle, \langle u' \rangle = 0, \ \ \langle X_t \rangle = ut, and\ \langle X_t^2 \rangle - \langle vt \rangle^2 = 2D_{L^t}\ D_l$ is the dispersion coefficient resulting from the flow heterogeneities [*Rinaldo et al.,* 1991;]. Under these assumptions, it has been observed that the probability $P(I;t)$ that the particle, found at the outlet at t,was at I at time 0 has a density which satisfies Kolmogorov's backward equation [*Rinaldo et al.,* 1991]. Its analytical expression in the case of a stationary process ($\text{i.e.}, v = \langle u \rangle$)constant throughout the network) is the inverse Gaussian distribution [*Rinaldo et al.,* 1991; equation (B5)]. It has also been observed that the model equation (53) may be consistently related to the basic equations of momentum and continuity in an open channel [*Rinaldo et al*., 1991;]. It is therefore (recall that$dx = vdt$) the case that

$$f(t) = \frac{\int_0^1 N_{eff(x,\Omega)}\, dP(x;t)}{Z(\Omega)} = \frac{\langle N_{eff}(vt,\Omega) \rangle v}{Z(\Omega)} \tag{54}$$

Where vt is the mean distance traveled in time t.

It is proper to compare (54) with prior concerned work. The link of channel network geomorphology via the width function is recognized [*Troutman and Karlinger,* 1985; *Gupta et al.,* 1986; *Gupta and Mesa,* 1988;]. In my notation, the width function formulation of the geomorphologic response is [e.g., *Gupta and Mesa,* 1988;. equation (17)]:

$$f(t) = \frac{vN_{eff}(vt,\Omega)}{Z(\Omega)} \tag{55}$$

I recognize that theequation (55) coincides with (54) in the simplest case $dP(x;t) = \delta(x - vt)dx$ (where $\delta(\ \ )$ is Dirac's delta distribution) of deterministic, purely convec-tive travel time distributions. In the case of uncertain velocity distribution, one expects wider bands on which the number of active states N_{eff} is to be averaged. In the geomorphological language this is another way of explaining the smoothing of the geomorphholological response in basins characterized by larger travel times even though the topological arrangement is self-similar at the different scales.

If the integral (54) is discretized in spatial steps of size $\varepsilon(\Omega) = 1/R_L^{\Omega-1}$ the following result is obtained exploiting the multifractal expression for N_{eff} in (43) and the inverse Gaussian distribution for $P(I;t)$;

$$f(t) = \frac{L(\Omega,\Omega)}{(R_B R_L)^{\Omega-1}\left(\pi D_{L} t^3\right)^{1/2}} \sum_{\gamma-1}^{R_L^{\Omega-1}} y\left(\frac{1}{R_L^{\Omega-1}}\right)^{\alpha_{m(y)}}$$
$$.\exp\left(-\frac{\left(2\gamma L(\Omega,\Omega)/\,R_L^{\Omega-1} - vt\right)^2}{4D_L t}\right) \tag{56}$$

with usual notation. To our surprise, (56) is similar to the geomorphological response derived by *Rinaldo et al.* [1991; equation (19)] by a completely different procedure. I observe that at small travel times and/or for momentum dispersion $(D_L \approx 0)$ the distribution of $P(x;t)$ degenerates into a Dirac delta distribution yielding the width function formulation of the instantaneous geomorphologic density unit which is then seen as a particular case of (56).
The following simple asymptotic expression holds for mean and variance of travel times [*Rinaldo et al.,* 1991;]:

$$E(T) = \frac{R_B - 1}{R_B}\frac{R_L}{R_L - 1}\frac{L(\Omega,\Omega)}{v} \tag{57}$$
$$Var(T) = \frac{2(R_B - 1)R_L L(\Omega,\Omega)}{R_B(R_L - 1)}\frac{D_L}{v^3} + \frac{(R_B - 1)R_L^2}{R_B^2(R_L^2 - 1)}\frac{L^2(\Omega,\Omega)}{v^2} \tag{58}$$

The geometrical contribution denotes the mean travel time, whereas the variance is affected by geomorphological dispersion. Elsewhere [*Rinaldo et al.,* 1991;] The geomorphological dispersion is bound to prevail in real basins.

It is interesting to apply the previous results to the Denwa basin, for which the following rules hold: $R_L = R_\varepsilon = 2, R_a = R_B = 4$ and $L(1,\Omega) = L(2,\Omega), L(\omega,\Omega) = R_L^{\omega-\Omega} L(\Omega,\Omega), with\ \omega > 2$ yielding

$$E(T) = \frac{3L(\Omega,\Omega)}{2v} \tag{59}$$
$$Var\ (T) = \frac{3D_L L(\Omega,\Omega)}{v^3} + \frac{L^2(\Omega,\Omega)}{4v^2} \tag{60}$$

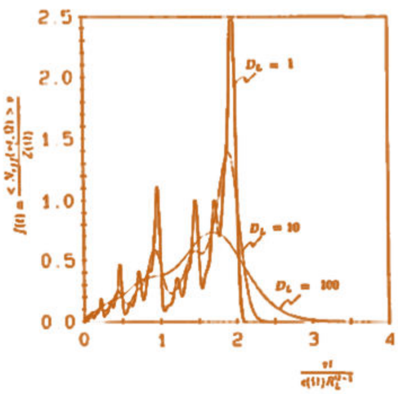

Figure. 5. The equation (56) for the Denwa basin computed for $N_{eff}(vt, 11)$ The scale of velocity is assumed as as $v = 0.5\ (L/T)$ in basin units. Values of D_L are I, 10, and 100 (L^2/T).

Equations (59) and (60) are assimilated with the asymptotic values of the moments computed by [*Troutman and Karlinger* 1985;] and discussed further by [*Gupta and Mesa* 1988;] for Dirac delta-distributed first-passage distributions ($D_L = 0$)in individual links and gamma-distributed link lengths in topologically random networks. In our notation Troutman and Karlinger's moments are:

$$E(T) = \pi^{1/2} \frac{L(\Omega,\Omega)}{v} \tag{61}$$

$$Var\ (T) = (4 = \pi) \frac{L^2(\Omega,\Omega)}{v^2} \tag{62}$$

because the (constant) link length in Denwa net is $L(1, \Omega) = L(\Omega, \Omega) R_B^{\Omega-1}$ and the magnitude is $R_B^{\Omega-1}$ The differences in the mean travel time are of the order of 10%. The larger variance of the geomorphologic contribution in the second formulation is due to the effects of the distribution of link lengths. Nevertheless, the results are clearly of the same order although derived by vastly different approaches.

To grasp some features of the equation (54) the function $f(t)$ is computed for Denwa basin. Figures 5 and 6 illustrate the equation (56) in the case of $\Omega = 11$. The range adopted for D_L in basin units is $1 \times 10^{-2} (L^2/T)$. In basin units the longest path (l_{max}) is unity, and a unit velocity yields travel from (l_{max}) to the outlet in unit time. I observe the smoothing of the hydrologic response at increasing values of D_L, suggesting the mutual dependence of the geometry of a network and the flow dynamics in shaping the character of the hydrologic response.

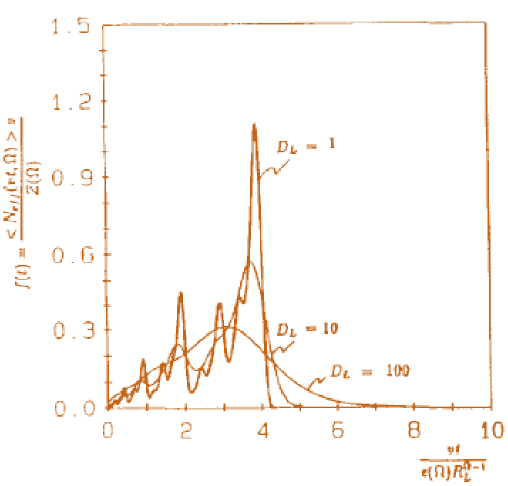

Figure. 6. The equation (56) for the Denwa basin computed for $N_{eff}(vt, 11)$ the scale of velocity is assumed as $v = 0.5\ (L/T)$ in basin units. Values of D_L are I, 10, and 100(L^2/T).

6. Conclusions

The study of geomorphological features of river basins has been presented and discussed. The total length, the fractal dimensions and the contributing areas have been linked and inroads to obtain a systematic geometrical description have been given. A connectivity conjecture is the limit of space-filling networks, the properties characterized by ratio of areas may be substituted by ratios of lengths of the network. Among the consequences are that the bifurcation ratio R_B equals, in the limit, the total area ratio R_A; the path probability in the momentum formulation is amenable to an analytic definition; and the topological distance from the outlet, generated by a multiplicative process in Hortonian networks, mimics the time evolution of total contributing areas. The relation of the geometry of fractal channel networks and their underlying geomorphologic response has been discussed in the framework of most probable momentum in a simplified dynamic context. The discussion about the multi-fractal characters of the dynamics of contributing areas seems of particular interest. An analytic expression for the arrival time distributions in fractal networks is derived and discussed in view of prior relevant work, most notably the width function formulation of the momentum. The latter is seen as a particular case of the most likely geomorphological response.

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