BED-LOAD MOVEMENT IN THE STEEP MOUNTAINOUS RIVER, DENWA

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Denwa River during different cycles, lowlands and troughs were developed respectively on the less resistant rocks and in the down-faulted blocks, which were usually bordered by high-level residuals of former erosion surfaces

Abstract - A method for the representation of bed-load data is given in this paper, in respect of steep mountainous river Denwa. The method is based on the conception that bed-load movement is the movement of bed particles, as governed by the laws of probability. By means of this method, an equation is obtained, which describes a great number of experiments in channels with uniform beds. A group of experiments conducted on sand mixtures, provides material for describing another application of the method.

INTRODUCTION

In normal course of denudation, the River Denwa has cut their way through steep mountainous tough Basalt flows into the underlying rocks. This process is naturally most rapid in the vicinity of the scarp, where erosion is greatest. This is partly due to the softness of the underlying rocks that the Denwa River is full of tough basalt boulders, which ground everything came into contact with them, as do ball in a ball mill. There are typical turbidity currents of a specific gravity of the flow, which is greater than that of the densest stagnant water in the basin. The high turbidity current of high density applied to current of sufficient density allow the Denwa River to carry a large bed load in suspension. At the bottom of the river slop, the flow possesses a large momentum and flows for a distant at a gradually decreasing velocity.

In the past, the problem of bed-load movement has been studied mostly by empirical methods. More recently, there has been a tendency to base movement studies on the theories of turbulence. (Hunter, Rouse. 1939, Benda, Lee, Marwan a. Hassan, Michael Church, and Christine 2005) It is the writer's belief that an approach to the problem of Movement can be made by a combination of the empirical and rational methods and that the results can be expressed by dimensionless plots.

Before proceeding with the development of these studies, it is necessary to discuss briefly two important considerations: (1) The difficulty or impossibility of defining, accurately, the so-called "critical" values; and (2) the possibility of correlating bed-load movement with local fluctuations in water velocity along the bed.

1. Attempts have been made in the past to derive an expression for the "initial movement" that is governed by certain definable "critical" conditions to be- used as the first step toward the solution of the Movement problem. In interpreting the results of many experiments on bed-load movement, and in comparing them with those obtained by other experimenters, the writer has concluded that a distinct condition for the beginning of movement does not seem to exist. It is just as impossible to determine the limit of initial movement as to determine the maximum possible flood of a river. Just as an expert is able to predict the probable maximum flood of a river to be expected within a given range of years, however, so is he able to define the hydraulic conditions in a stream, that will produce any given small rate of movement, which might be called the limit. This value can be chosen without any restriction. It is difficult to believe that the hydraulic conditions that will produce such movement could have any special meaning in the problem of movement as a whole. Therefore, the writer will not use the conception of critical tractive force, or any other critical value, when the term "critical" pertains to the flow, where movement begins.
2. In general, movement of bed load has been described as follows:' a particle of the bed moves when the pushing force or lifting force of the water overcomes the weight of the particle. This push or "lift" is expressed in terms of the average flow (William ,W. Rubi ,1938, Banziger, R. & Burch, H. 1990). The usual conception is that, movement begins when the velocity increases enough to overcome the weight, and that, with further increasing velocity of the water, the rate of movement will also increase, following a certain law that is found empirically. To prove that this conception is misleading, assume that the force acting on a particle could be described by means of the average flow alone. If the velocity of the water is increased gradually to a point at which the first particle would just be moving, the force acting on all the other particles of the same kind and size would move those too. Therefore, in a uniform bed, where all particles have the same size and shape, all would start moving together; they would be unable to settle again because at all points the water velocity is just sufficient to start movement. If this is true, there could be no law governing the rate of movement, but only a critical velocity. At all under critical velocities, there would be no movement, whereas, at all supercritical velocities, the rate of movement would be limited only by the number of particles available (Yager, Em . 2007 , Herbert Lang, A. Musy1990). Therefore, it cannot be presumed that the rate of bed-load movement is a function of the average flow. Instead it is proposed to express it in terms of the fluctuations of the water velocity near the bed (MarioAristide Lenzi, Luca Mao, Francesco Comiti 2006 , Chiari ,M . 2011).

Results of previous studies (Wasserbau, Hydrologie und Glaziologie. Ethz. Nr. , Zurich Rickenmann, D., 1990) describing the movement of a bed-load particle by means of statistical methods are to be used in an attempt to coordinate the rate of movement with the fluctuations of the water velocity near the bed. The results of these studies can be summarized as follows:

(a) These flume studies dealt with the movement of rather coarse particles along a bed consisting of the same kind of grains. Being coarser than 2 cm. in these particles always remained near the bed, rolling, sliding and, sometimes, according to the normal description of bed-load movement. It was found that the moving bed load and the bed on which it was moving formed a unit, in as much as there was a steady and intensive exchange of particles between the two. Thus it is concluded that all the particles of the bed, down to a certain depth, take equal part in the movement, alternatively moving and returning into the bed (Warburton ,Jeff 1992).

(b) Bed-load movement is to be considered as the motion of bed particles in quick steps with comparatively long intermediate periods of rest. Thus bed-load movement is a slow downstream motion of a certain top-layer of the bed (Hydraul, J. Eng. 2008).

(c) The average step of a certain particle seems always to be the same even if the hydraulic conditions or the composition of the bed changes; and

(d) Different rates of movement are produced by a change in average time between two steps and by a change in the thickness of the moving layer.

These concepts permit the development of a formula in general terms. The rate of movement will be described by means of this average "step."

DERIVATIONS

This paper will treat only the bed-load movement of uniform sediment and mixtures acting like uniform sediment. In both cases it is possible to describe the sediment by a representative diameter D and its density ρ_s. The expression "acting like uniform material" means that both bed material and moving material have the same composition and, therefore, the same representative diameters. It is possible in this case to describe movement at a certain point of the bed by one symbol; namely, the rate of movement q_s.(1) Bed material moving in suspension; and (2) bed-load movements, in which the composition of the bed is essentially different from the composition of the moved material.

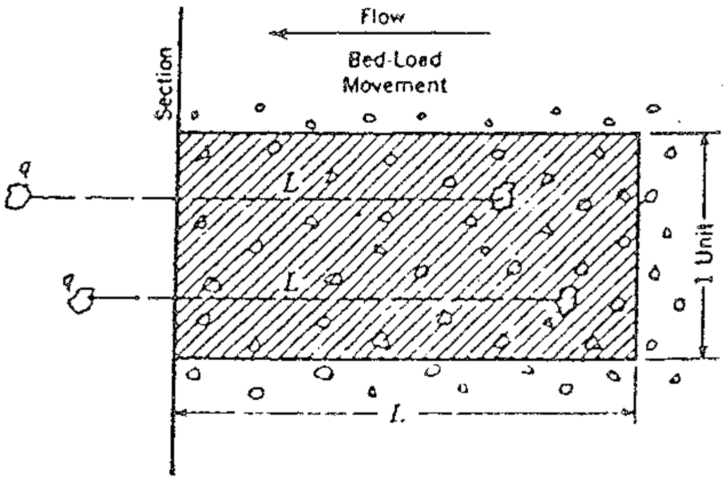


Figure 1

A bed-load formula is an equation linking the rate of bed-load movement with properties of the grain and of the flow causing the movement formula of this kind can be derived by expressing in an equation, the fact that all particles passing the unit width of a section as bed load are just on the way to perform one of these steps of the constant length $L = \lambda_0 D$. Figure.1 shows the cross section and the rectangular area with the length L and with unit width, where all particles start the steps that together form the rate of movement q_s .Eq. 1 expresses the condition that the total volume passing the unit width of the section per second (q_s divided by the specific gravity of the particles, both under water) equals the total volume of all the particle starting a step during a second in the aforementioned rectangular area. This volume is obtained by multiplying the number of particles in the surface of the area by the probability that a particle in the bed surface will start moving during a given instant and with the volume of a given particle. Eq. 1 follows:

$$\frac{q_s}{(\rho_s-\rho_f)g} = \frac{L}{A_1 D^2} p_s A_2 D^3 = \frac{A_2}{A_1}\lambda_0 p_s D^2, \qquad (1)$$

In which: q_s = the rate of Movement, in weight (under water), per unit of width, per second;
ρ_s and ρ_f = density of particle and fluid, respectively;
g = acceleration due to gravity;
D= representative diameter of the particles;
A_1 and A_2 = dimensionless ratios such that A_1D^2 = the area that the grain covers in the bed and A_2D^3 = the volume of the particle; p_s = the probability that a particle will start moving in any given second; and λ_0 the dimensionless measure for the length of the single step.
It must be kept in mind, however, that λ_0 may or may not be a constant; that is, and it has not been proved to be constant.

If *A* and *D* are transposed to the left side of Eq. 1, that side will include all terms pertaining to the grain, whereas the right side $\lambda_0 p_s$is still an unknown function of the flow-that is:

$$\frac{q_s}{(\rho_s-\rho_f)gD^2}\frac{A_1}{A_2} = \lambda_0 p_s, \ \ldots\ldots\ldots\ldots\ldots \qquad (2)$$

In this equation A_1, A_2 and λ_0 are dimensionless, but p_s has the dimension sec^{-1}. In order to make the right side of Eq. 2 dimensionless p_s must be multiplied by a given time. The most reasonable time to use is the average time, t, required for the water to remove one particle from the bed. If $p = tp_s$ then p is dimensionless and gives the number of steps that start from any given place during the time it takes to remove one particle. The maximum value of p is 1, and indicates that at all times and at all points particle by particle starts to move. The minimum value of p is zero; therefore, p expresses the probability that a step is about to begin at a given place. These steps start everywhere on the bed. Therefore, p can be interpreted as the probable part of the bed area in which steps are starting. A step is started only at a point where the hydraulic lift of the water is able to overcome the weight of the particle. Therefore, p expresses the probability that the hydraulic lift on any particle along the bed is about to overcome the weight of the particle.

Unfortunately there is no method of expressing or measuring the time t required for the lifting force to pick up a particle. It is assumed to be proportional to some other characteristic time of the particle in .the water. The time that the particle requires to settle in water, a distance equal to its own diameter D, is chosen for this characteristic. The reason for choosing this time was the fact that it is the only expression with the dimension of a time, which is representative for the behavior of the particle in the liquid without including any characteristic of the flow. This time can be expressed as

$$\frac{D}{v_f} = \frac{1}{F}\sqrt{\frac{D\rho_f}{g(\rho_s-\rho_f)}} \ldots\ldots\ldots\ldots\ldots \qquad (3)$$

In which: v_f = the velocity of a particle settling in water; and F = parameter for settling velocity. In Equation *3, F= 0.816* for particles greater than 1 mm, settling in water of normal temperature.Fig.2 shows the values of F for smaller grain sizes. Characteristics of the materials in Fig. 2 are Kinematic viscosity $v = \frac{\mu}{\rho_f}$ equals, for water, 0.012, and, for air, 0.16, $cm^2 per\ sec$; and specific densities, $\frac{\rho_s-\rho_f}{\rho_f}$. are as follows-

Material	Specific *density*
Barite in water	3.22
Gravel in water	1.65
Soil in water	0.25
Gravel in air	2,210.0

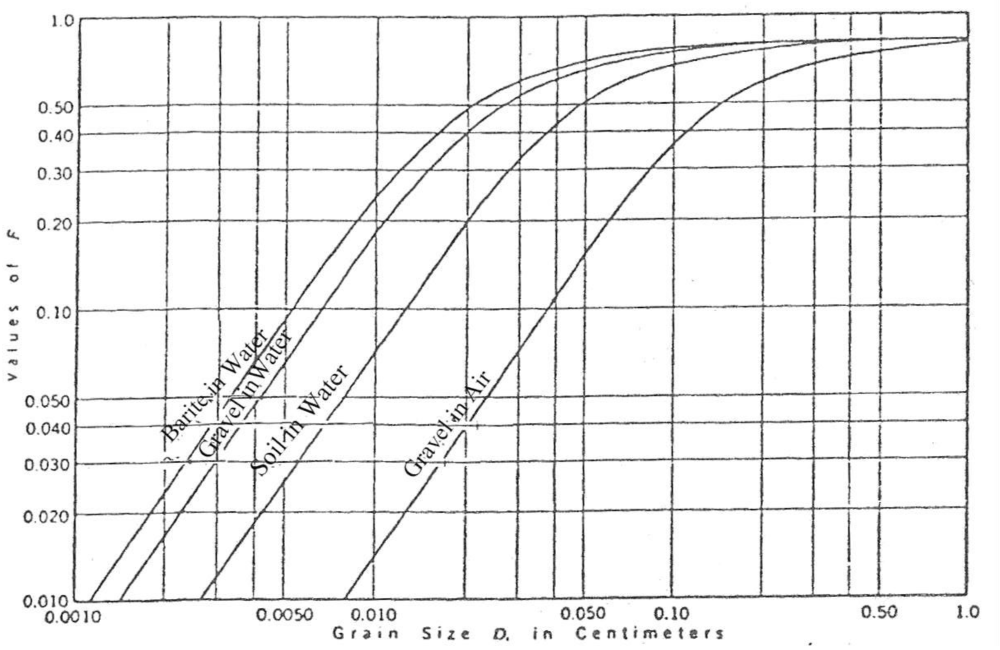


Figure: 2 Parameter *F* for determining the settling velocity of various materials

In Fig. 2 the following equation for the settling velocity derived by William W. Rubey has been used for the determination of *F*:

$$v^f = \sqrt{\frac{2}{3}\frac{\rho_s-\rho_f}{\rho_f}D + \frac{36\mu^2}{\rho_f^2 D^2}} - \frac{6\mu}{\rho_f D} = F\sqrt{Dg\frac{\rho_s-\rho_f}{\rho_f}} \ldots\ldots\ldots\ldots\ldots \quad (4)$$

In this formula all terms are measured in centimeter-gram-second units. Hence, F will be

$$F = \sqrt{\frac{2}{3} + \frac{36\mu^2}{gD^3\rho_f(\rho_s-\rho_f)}} - \sqrt{\frac{36\mu^2}{gD^3\rho^f(\rho_s-\rho_f)}} \ldots\ldots\ldots\ldots\ldots \quad (5)$$

The time t required to remove a particle from its place in the bed then will be

$$t = \frac{A_3}{F}\sqrt{\frac{D\rho_f}{g(\rho_s-\rho_f)}} = \frac{p}{p_s} \ldots\ldots\ldots\ldots\ldots \quad (6)$$

in which A_3 is still an unknown constant. Eq. 1 can now be changed to the form

$$p = \frac{A_1 A_3}{\lambda_0 A_2}\left[\frac{1}{F}\frac{q_s}{(\rho_s-\rho_f)g}\sqrt{\frac{\rho_f}{\rho_s-\rho_f}}\frac{1}{g^{0.5}D^{1.5}}\right] \ldots\ldots\ldots\ldots \quad (7)$$

In an attempt to express p as the probability of the local hydraulic lift to overcome the weight of the particle, p refers to the part of the bed in which locally (at a certain moment) the lifting force is greater than the weight of the particle. It can be stated that p refers to the part of the bed in which the ratio of the local lift to the average lift is greater than the ratio of the weight of the particle to the average lift. In mathematical terms this is

$$p = f\left(\frac{Weight\ of\ the\ partical}{Average\ life\ of\ the\ particle}\right) \ldots\ldots\ldots\ldots \quad (8)$$

in which is an unknown function. The weight of the particle under water is $A_2 D^3(\rho_s - \rho_f)g$,and the average lift is

$$\text{Lift} = A_4 D^2 v^2 \rho_f \ldots\ldots\ldots\ldots \quad (9)$$

v being a local velocity at some still unknown distance from the bed. An approximate value for v is:

$$v = 11.6\sqrt{\frac{\tau}{\rho f}} \ldots\ldots\ldots\ldots \quad (10)$$

Eq. 10 may need to be corrected in the future. It defines the $velocity^3$ at the edge of the laminar boundary layer if the wall is smooth, or the velocity at the distance D if the wall is rough, in which D is a measure of average roughness. The shearing stress τ along the wall is

$$\tau = \mathrm{SR}\rho_f g \ldots\ldots\ldots\ldots \quad (11)$$

in which S is the slope and R is the hydraulic radius; therefore

$$v = 11.6\sqrt{SRg} \ldots\ldots\ldots\ldots \quad (12)$$

Eq. 8 can now be written

$$p = f\left[\frac{A_2 D^3(\rho_s-\rho_f)g}{(A_4 D^2 \rho_f)(135\, S\, R)g}\right] = f\left[\frac{A_2}{135 A_4} \times \frac{(\rho_s-\rho_f)D}{\rho_f\, S\, R}\right] \ldots\ldots\ldots \quad (13)$$

By assuming that Eq. 10 gives the correct value for the velocity, Eqs. 7 and 13 can be combined and a new movement formula formed:

$$A\left\{\frac{1}{F}\left[\frac{q_s}{(\rho_s-\rho_f)g}\right]\sqrt{\frac{\rho_f}{\rho_s-\rho_f}\frac{1}{g^{0.5}D^{1.5}}}\right\} = f\left[B\left(\frac{\rho_s-\rho_f}{\rho_f}\frac{D}{S\, R}\right)\right] = p. \quad (14)$$

in which

$$A = \frac{A_1 A_3}{\lambda_0 A_2} \ldots\ldots\ldots\ldots \quad (15a)$$

and

$$B = \frac{A_2}{135\, A_4} \ldots\ldots\ldots\ldots\ldots\ldots \quad (15b)$$

are constants which, however, may vary with different shapes of the particles. Whether λ_0 and A_4 really are constant under all conditions must be determined later. Introducing

$$\emptyset = \frac{1}{F}\frac{q_S}{(\rho_s-\rho_f)g}\sqrt{\frac{\rho_f}{\rho_s-\rho_f}\frac{1}{g^{0.5}D^{1.5}}} \ldots\ldots\ldots\ldots \quad (16a)$$

and

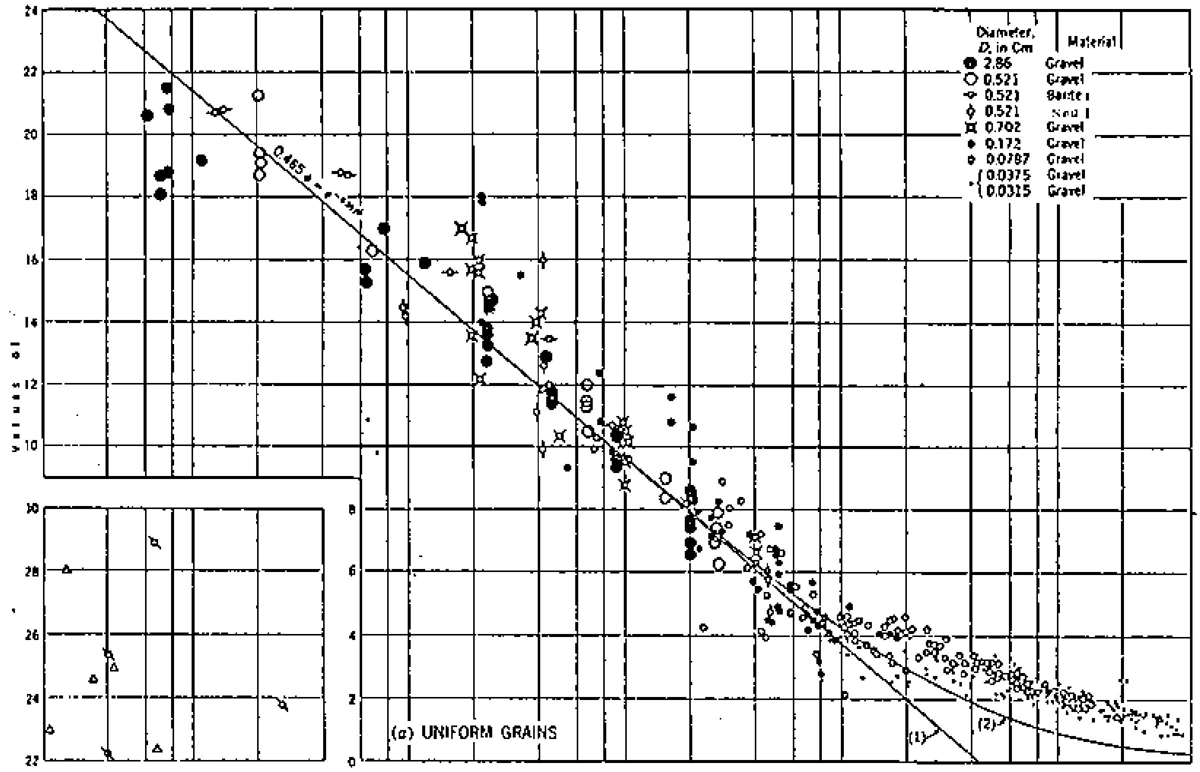
$$\psi = \frac{\rho_s-\rho_f}{\rho_f}\frac{D}{S\, R} \ldots\ldots\ldots\ldots\ldots \quad (16b)$$

Eq. 14 can be written in the short form:

$$A\phi = f(B\psi) = p \ldots\ldots\ldots\ldots\ldots \quad (17)$$

The function f as well as the two constants A and B must be determined empirically. Data from a great number of measurements using various materials have been analyzed, and values of ψ

and Ø computed. A semi logarithmic plot of these values is shown in Figure. 3(a). The grain sizes range
from 0.315 to 28.65 mm in diameter, the water depth from 18 to 1,100 mm, and the specific gravity of the particles from 1.25 to 4.22. All these experiments are performed in flumes with uniform sediment. The experiments conducted by Zurich and Meyer-Peter, E 1934 are described briefly in this paper. Whereas, the remaining experiments are taken from the generally known paper by (G. K. Gilbert, 1914). It seems that all these points follow, reasonably, a single curve. It might be mentioned also that the hydraulic radius R is computed by a method by (Clifford R. Blizard, Ellen E. Wohl 1997) that eliminates the effect of side-wall friction and gives results comparable to a channel of infinite width.



In Fig. 3(a) all the points with values of ϕ less than 0.4 seem to follow the straight line, curve (1), the equation of which is

$$0.465\phi = e^{-0.391\psi} \ldots\ldots\ldots\ldots \quad (18)$$

If Eq. 18 is assumed to represent the law of Movement:

$$\left.\begin{array}{l} A = 0.465 \\ B = 0.391 \\ f(x) = e^{-x} \end{array}\right] \ldots\ldots\ldots \quad (19)$$

It remains only to explain why the points $\phi > 0.4$ seem to be too high. If Eq. 18 is accepted as a general law, the value $\phi > 2.15$ would not be possible because p cannot exceed 1. Therefore, values $\phi > 2.15$ are possible only if A becomes smaller (A consists of the constants A_1, A_2, A_3 and λ_0).
The constants A_1, A_2 and $A_3$are not likely to change with increasing values of p, but λ_0 does. The distance λ_0 has been found to be constant for small values of p—that is, when the hydraulic lift seldom exceeds the weight. As p increases, it more often happens that, in the very spot where the step would have ended, there exists a local lift strong enough to keep the particle from settling. The oftener this happens the more λ_0 seems to increase on the average. The symbol p expresses the probability

that the lift exceeds the weight of the particle for every point on the bed. Therefore, only $(1 — p)$ particles of the unit will be able to settle after a step λ_0. The other p particles will start for another λ_0, and out of these $(1—p)\,p$ will settle after the second λ_0,'and so on. The average distance traversed by the unit, therefore, is:

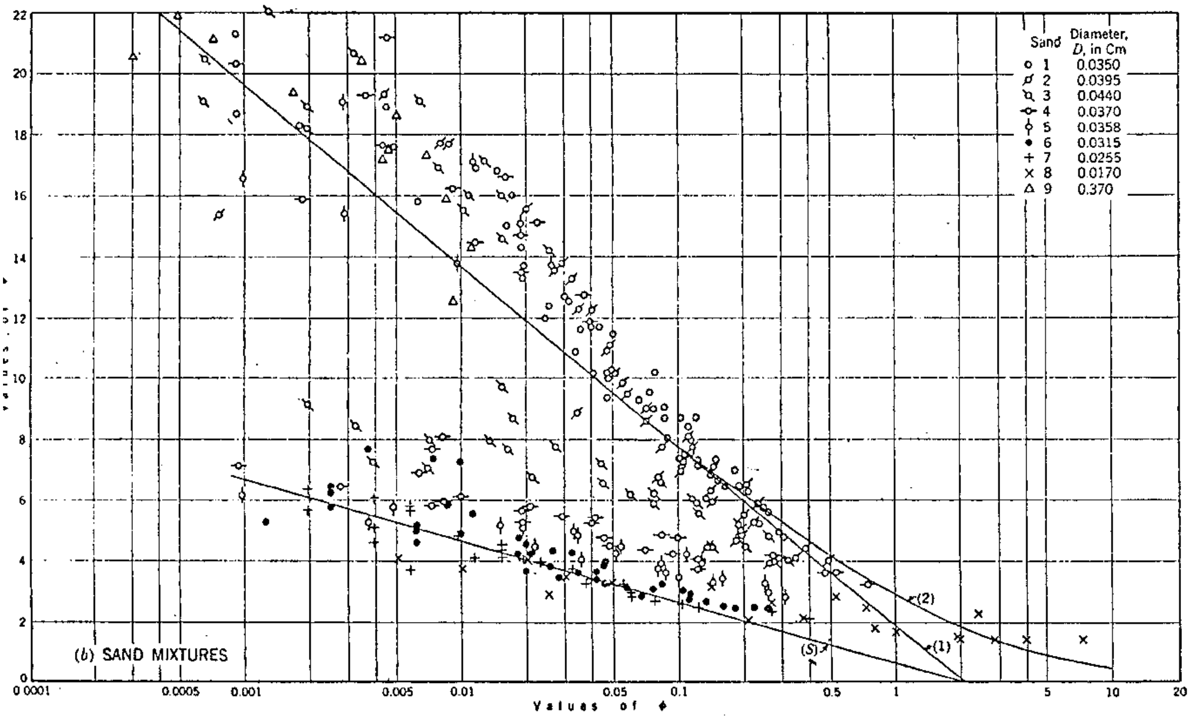


Figure: 4. Denwa River, Bed –Load experiments showing the relation between ∅ and ↓

$$\lambda = \sum_{m=o}^{\infty}(1-p)p^{m-1}m\,\lambda_0 = \frac{\lambda_0}{1-p}\,.........\qquad(20)$$

in which $m = a$ whole positive number. Introducing λ instead of λ_0 yields curve (2) instead of curve (1). This new curve follows the plotted points more closely than curve (1).

The constant A_4 for the lift in Eq. 9 is introduced without further discussion. The question arises whether the deviation of the points from curve (2) could be due to a change in the constant A_4. Constant A_4 could change only with the Reynolds number of the flow around the particle, or with the value of $\frac{D}{\delta}$, the ratio of the grain size to the thickness of the laminar layer. Eq. 10 gives the local velocity of the water:

$$v = 11.6\sqrt{\frac{\tau_0}{\rho_f}} = 11.6\sqrt{S\,R\,g}\,..........\qquad(21)$$

The Reynolds number of the local flow is

$$R = \frac{D\,u}{v} = \frac{11.6\,D\sqrt{S\,R\,g}}{v}\,...........\qquad(22)$$

and the thickness of the laminar layer is

$$\delta = v\frac{D\,v}{\tau_0/\rho_f}\,...........\qquad(23)$$

$$\frac{D}{\delta} = \frac{D\sqrt{S\,R\,g}}{11.6v} = \frac{R_D}{134}\,...........\qquad(24)$$

As $\frac{D}{\delta}$ differs from R_D only by a constant factor, it is sufficient to study the influence of only one of them. The deviation of the measured points from curve (2) Plotted against D/δ failed to disclose any satisfactory relationship. Each grain size appears to follow a separate curve; therefore, it appears much more probable that A_4 is a constant, but that the exponential law for p does not extend down to $\psi = 0$. Another explanation for the deviation of the points may be that part of the grains, have been moved in suspension. In this case those experiments would be outside the field of application of Eq. 17. It is emphasized that in most Pachmarhis rivers, the bed load is largely transported under conditions pertaining to this part of the curve.

The title of this paper was chosen specifically to avoid the impression that any attempt was being made to discover "the law of bed-load movement," because it is the writer's belief that such universal law does not exist in a simple mathematical form. Just as it is necessary to distinguish between friction in smooth and rough pipes or channels, so it is necessary to distinguish between different kinds of movement. Nevertheless, the distinction between friction along rough, wavy, and smooth walls was only possible on the discovery of the general method of plotting the friction factor against Reynolds' number. An attempt is made in this paper to determine a corresponding method of presenting movement data by introducing the quantities ψ and ϕ, both of which are derived by pure speculation, using only generally known facts.

As an example, the method is used to discuss the results of experiments with sand mixtures, conducted at the laboratory of Tawa Dam Tawanagar (M.P.). Figure. 3(b) gives the results of these experiments as a $\phi - \psi$ graph. Curves (1) and (2) are transferred from Figure. 3(a), and all the various sand mixtures have been assigned different symbols.

The first problem was to determine the effective diameter of the mixtures— that is, the value of D that would represent the mixture in the formulas. Experience gained in previous studies has convinced the writer that the most usable value for this effective diameter is the grain size of which 35% to 45% of the material is finer. This value is readily obtained from the cumulative size-frequency curve of the mixture. The 40% value was used for Figure. 3(b), although it is realized that the use of a 35% value would tend to bring the high points closer to curves (1) and (2).

The distribution of the points in Figure. 3(b) is very interesting to note. At first glance one observes that the points for sands 1, 2, and 9 distinctly follow curves (1) and (2). Sands 3, 4, and 5 follow the curves in the upper part only. Sands 6, 7, and 8 fall below the curves at all points, but a distinct grouping of points along a line curve (5) is noticed.

The two curves, (2) and (S), seem to represent limits of maximum and minimum Movement for a given value of ψ. In searching for an explanation of this the three following questions naturally arise: (1) is there any relationship between the position of the points and friction loss? (2) Is there any relationship between the position of the points and the condition of the bed?

(3) Would a similar distribution be possible also in experiments with uniform material, or is it characteristic only of mixtures?

With regard to question (1), it was found that Manning's n, without exception, increased suddenly when the points leave curves (1) and (2). This means that the bed becomes rougher than the original material as soon as the rate of movement decreases below that shown by curves (1) and (2), The reverse is also true—that is, the rate of movement will decrease as soon as the roughness of the bed increases.

The reason for this increased roughness is of interest. As a rule, riffles begin to form precisely at the place where the points depart from curve (1). If riffles are the reason for the increase in Manning's n, this value must always increase when riffles are formed. Sand 1 does not show this increase and sand 2 only very slightly—although these sands develop general riffles like all the other mixtures. Therefore, the riffles are not the reason for the increased roughness, but merely happen to develop simultaneously. This answers question (2) in the negative.

Question (3) suggests that perhaps some kind of sorting of the grains is the reason for the deviation from curve (1) and for the increase of roughness at the same time. It is the writer's belief that this is true, but unfortunately it is not subject to direct proof. This sorting would be caused by the lack of material in the upper end of the flume. If the sand is fed in at a smaller rate than the stream is able to transport it, the bed starts to build some kind of a protective layer of coarse grains on the surface and buries all the finer grains beneath. The average grain size in this coarse layer is much

greater than the average grain size in the bed, and scour is either reduced or completely prevented. For this reason, it is impossible, during an experiment, to detect a lack of feeding merely by watching the position of the bed. Curve (1) gives the results obtained when the highest quantity of sand is fed in that can be transported without deposition, and curve *(S)* gives the smallest amount that will be transported without scour on the protecting layer. If D is the effective diameter of the original bed material, it is reasonable to believe that these two limits coincide for uniform material and that curve *(S)* falls more and more below curve (1) as the material decreases in uniformity.

These are merely some suggested methods of studying bed-load Movement. (Benda, lee, Marwan a. Hassan, Michael Church, and Christine 2005 , Yager, Em. 2007 , Herbert Lang, A. Musy1990) It would be very easy to determine, by experiment, the validity of such reasoning. If the interpretations are correct, it should be possible to determine all points between the two limiting curves by merely changing the rate of sand feed. It would also be very instructive to conduct a similar group of experiments in the opposite sequence—that is, by beginning with high discharges and progressively decreasing the discharge and load. If the explanation is correct, one will probably not return to the same curve obtained with increasing flow. This would also answer question (3). This $\phi - \psi$ method is offered as a new procedure for studying bed-load problems. It may be possible to refine the method by introducing corrections for the velocity *v*, and various constants; but as a whole it seems to be satisfactory in its present form.

CONCLUSION

In concluding, it may be stated that the treatment of movement problems by means of statistical methods, made possible by the use of large-scale experiments, led to the proposed method of representation:

Two dimensionless functions ψ and ϕ have been developed theoretically, ψ as the ratio of the forces acting on the particle, and ϕ including the rate of movement and the size and settling velocity of the particle. The interrelation between these two functions expresses the law of movement, and at the same time expresses the statistical distribution p of the velocity of the liquid close to the laminar boundary layer. The movement law is derived by use of a great many experiments with uniform sediment, and is then used in discussing published results of experiments with sand mixtures.

REFERANCES

- Aronne Armanini, Gianpaolo Di Silvio (1991) Fluvial hydraulics of mountain regions, Springer-Verlag USA Geol. Sur.
- Banziger, R. & Burch, H. (1990) Bed load transport in a mountain torrent. IAHS Publ. no. 193: 207-214. USA Geol. Sur.
- Benda, Lee, Marwan A. Hassan, Michael Church, and Christine L. (2005), Geomorphology of steepland headwaters: the transition from hillslopes to channels, *J Am Water Resources Assoc*, *41*(4), 835. USA Geol. Sur.
- Clifford R. Blizard, Ellen E. Wohl (1997) Relationships between hydraulic variables and bedload transport in a subalpine channel, Colorado Rocky Mountains, U.S.A. Department of Earth Resources, Colorado State University, Ft. Collins, CO 80523 USA.
- Chiari M – (2011) Calculation of bedload transport in steep channels with a numerical model Earth surface processes and Landforms 36, 805-815 USA Geol. Sur.
- Ergenzinger, Peter; Schmidt, Karl-Heinz (1994) Dynamics and Geomorphology of Mountain Rivers , Lecture Notes in Earth Sciences, Volume 52. ISBN 978-3-540-57569-6. Springer-Verlag, USA Geol. Sur.
- Erich R. Mueller and John Pitlick.(2005) Morphologically based model of bed load transport capacity in a headwater stream. Department of Geography, University of Colorado USA.
- Gilbert G.K., (1914) The Movement of Debris by Running Water. Professional Paper No. 86, Geological Survey, Washington.
- Gomez, B., Church, M., (1989): An Assessment of Bed Load Sediment Transport Formula for Gravel Bed Rivers. Water Resources Research, Vol. 25, No. 6: 1161 - 1186.
- Gintz, D., Hassan, M. A. and Schmidt, K.-H. (1996), Frequency and magnitude of bedload transport in a mountain river. USA Geol. Sur.
- Herbert Lang, A. Musy (1990) Hydrology in Mountainous Regions: Hydrological measurements. The water cycle International Association of Hydrological Sciences Northwest Watershed Research Center, 270 South Orchard, Boise, Idaho 83705, USA.
- Hegg, Ch., Rickenmann, D., 1998: Short-time relations between runoff and bedload transport in a steep mountain torrent. IAHS Publ. No 249: 317-324. USA Geol. Sur.
- Hegg, C (1999) Comparison of bedload transport in a steep mountain Swiss Federal Institute for Forest, Snow and Landscape Research Birmensdorferstr. 111, CH-8903 USA.

- Hydraul, J. Eng. (2008); Bed-Load Transport Flume Experiments on Steep Slopes Journal of Hydraulic Engineering / Volume 134 / Issue 9 USA.
- Meyer-Peter E. Favre, H. and Einstein, (1934) Nauere Versucharesulate ober den Geschiebetrieb, Schwoizerische Bauzeitung, Vol.103 No. USA Geol. Sur.
- Mario Aristide Lenzi, Luca Mao, Francesco Comiti (2006) When does bedload transport begin in steep boulder-bed streams? Volume 20, Issue 16, Article first published online: 24 APR 2006
- Rickenmann, D., (1990): Bedload transport capacity of slurry flows at steep slopes. 103, Zurich. USA Geol. Sur.
- William W. Rubey, D. C., (1838). The Force Required to Move Particle on a Stream Bed, Professional Paper No. 183-E, U. S. Geological Survey, Washington.
- Washington, D. C, J, (1939) An Analvsis of Sediment Trarsportntion in the Light of Fluid Turbulence, , Sedimentation Dir., SCS-TP-25, USA Geol. Sur.
- Warburton ,Jeff (1992) Observations of Bed Load Transport and Channel Bed Changes in a Pre glacial Mountain Stream Arctic and Alpine Research Vol. 24, No. 3 USA Geol. Sur.
- Yager Em – (2007) Calculating bed load transport in steep boulder bed channels water resources research, vol. 43, USA Geol. Sur.