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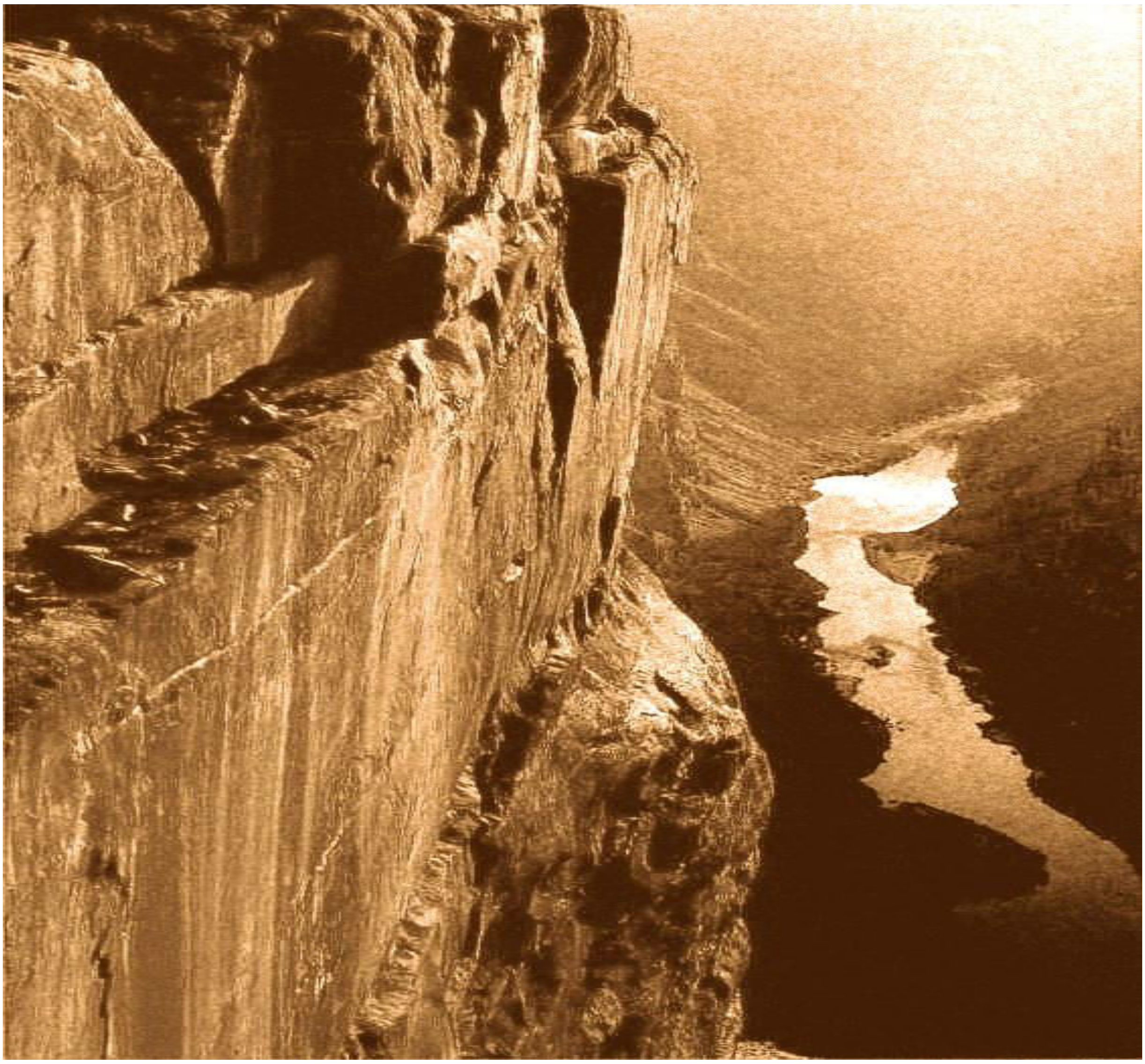
RATIONAL DERIVATION OF RIVER-BED PROFILE



The Sonbhadra Gorge

Abstract - The Denwa, Sonbhadra, Bainganga, and Nagdwari rivers, in the Pachmarhi and Rift Valleys substantiate the premise of this paper, that wear-phenomena offer a rational approach to river-bed-profile morphology. The Inference is that the bed-load plays a very important role in forming the river-channel. The caution is Indicated, the rational instrument offered here for the analysis of tectonics of the river- profiles gives a practical means for expressing the vertical shape of a river-bed so that the morphologic effect of man-made measures can be studied. The usefulness of $S = S_{0^{e^{ax}}}$ and $Z = (S_{0/a})\,(e^{ax} - 1)$ lies in the fact that they formulate the equilibrium-profile, that which will prevail ultimately or after complete morphologic development. Hence, if the regimen of a river is altered by regulation, a practical prediction of the final form can be ventured, with sore rational basis for the estimate than has been the case hitherto.

A fanciful possibility is the establishment of a relation between an overall wear-coefficient like "a" and the River Tectonics that should permit fluvial or physiographic predictions In a manner akin to the use of flood-formulas-A, certainly an accuracy equal to the dubious one of this common hydrologlic tool, should be attainable.

It is indeed true, that valleys of the Pachmarhis are often seen to follow tectonic lines: trough, faults, and rock–cliff. There are tectonic lines coincides with alignment of a valley, that proves it has influenced the course the river has taken.

River Morphologist has been faced for many years with the dilemma that on the one hand, its accumulated experience is sometimes inadequate for reliable prediction of river behaviour while, on the other hand there has been little attempt at systematic quantitative evaluations to 'discover identity in difference' all the

above referring particularly to the form of river-channels, in cross-section plan and profile. The river bed is also a part of structure, so that the purpose of this paper is to test the structure features and review certain principles which afford a rational derivation of the profile of river-beds, results on Denwa River. (Tributary of Tawa)

Derivation of equation of river-bed profile

As particles of bed-load move downstream, their size or weight is reduced by abrasion or wear, collision and solution. The most important of these factors is abrasion, for which Sternberg (1933) developed a law that has been verified experimentally by Schoklitsch (1938). The reduction in weight of a stone or particle as it travels downstream should be proportional to the work done against friction along the bed. If P is the weight of the stone and φ the coefficient of friction, the frictional resistance to motion is at any instantφP. The work against friction in a very short distance dx is φPdx. Let dP be the resulting decrease in weight. Then it can be said that

$-dP \ = \ c\varphi Pdx,$ (1)

in which c is a constant of proportionality. The minus sign indicates that P decreases as x increases that is $-dP$ goes with $+dx$ Integration yields

$P = P_0 e^{-ax},$ (2)

$\log_{10} P = \log_{10} P_0 - ax \log_{10} e = \log_{10} P_0 - 0.434ax,$ (3)

With $c\varphi$ replaced by "a" and $\log_{10} e$ by 0.434. In this, the Sternberg abrasion law, P is the weight of a stone in kilograms after traveling a distance of x kilometer downstream from the starting point where the initial weight was P_0 while e is the base of natural logarithms. The quantity "a" is called the coefficient of abrasion or wear of the bed-load material and represents the loss in weight of a stone weighing one kilogram after traveling one kilometer; the units of "a" are kilograms per kilogram per kilometer or kilometer^{-1}. It might just as well be called the coefficient of weight decrease a significance that will be recalled below.

Saxena, (1993) measured the average pebble-size in along the Denwa river and found empirically that the size-variation with distance along the Denwa river followed an exponential law like that in equation (2).

Since the bed-slope of a river and the size of bed- load material are known to decrease from source to mouth, it is not far-fetched to assume the slope proportional to the size of bed-load material. Mathematically expressed

$S = kP_0 e^{-ax},$ (4)

where S e is the bed-slope and k is another constant of proportionality. At the starting point where $x = 0$,$S = S_0$, so that $kP_0 = S_0$ and

$S = S_0 e^{-ax},$ (5)

$\log_{10} s = \log_{10} s_0 - 0.434ax,$ (6)

This is the equation of the slope of river profiles. A more detailed derivation of equations (2) and (5) will be found in the work of Shulits.

$S = S_0 e^{-ax}$ is a rationally deduced formula, based on reasonable or at least plausible assumptions and not on measured data. There is no compulsion for a measured, actual bed-profile to conform to it. But if profiles can be shown to have an equation of this form the inference must be that equation (5) is reliable and valid. Proof of this will follow below.

Application to practical problems has shown that a slight variation in the form of above slope-distance equation is convenient. It is desirable to let S_0 be the slope at a downstream initial point and to measure x positive upstream from S_0 , where $x = 0$. Equation (5) then becomes

$S = S_0 e^{ax},$ (7)

The simple mathematical transformation need not be given here. Fig 1 is a schematic representation of the significance of the variables for this equation and another to be derived later.

The above formulas are dimensionally correct so that any consistent system of units may be used. In this paper the following will be the system of units: S in meter per kilometer, x in kilometer. and "a" in kilogram per kilogram per $kilometer$ or $kilometer^{-1}$.

Validity of $S = S_0 e^{ax}$,—Equation (7) can be written

$\log_{10} S = \log_{10} S_0 + ax \log_{10} e,$ (8)

Whence it is evident that, $S = S_0 e^{ax}$, should plot as a straight line on semi-logarithmic paper which is the clue to the proof of its reliability. If the data from an actual river yield a linear relation on semi-logarithmic paper, then $S = S_0 e^{ax}$, is justified.

The proof has another important aspect. From (8) can be obtained.

$0.434a \ = \ (\log_{10} S - \log_{10} S_0)/x,$ (9)

But the right-hand side of the above is the slope of the straight line on semi-logarithmic paper, as Fig 2 shows; and the wear- coefficient, a, is then the slope of the line divided by 0.434

$a = (\log_{10} S - \log_{10} S_0)/0.434x = \log_{10}(S/S_0)\ /0.434x,$ (10)

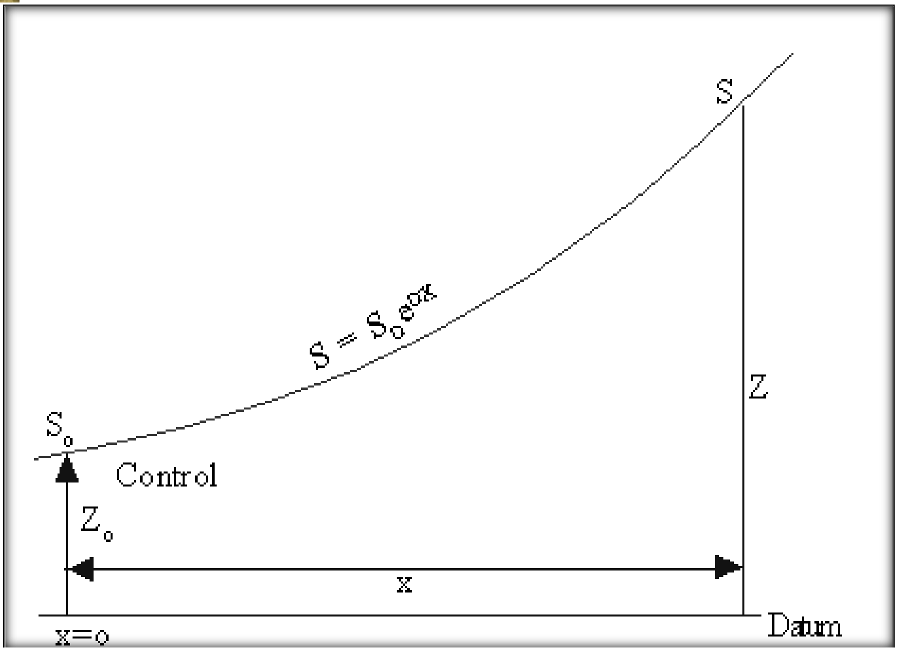


Figure 1. Variables in bed-slope equation $S = S_0 e^{ax}$

Accordingly if the wear-coefficient so determined from the plotted data for an actual river corresponds to the kind of material of which the bed-load is known to consist a second test of validity has been made. By writing (10) thus,

$$\log_{10}(S/S_0)\ /0.434x, \tag{11}$$

it can be seen that the logarithm of the ratio of slopes is proportional to the loss in weight suffered by a one-kilogram stone in traveling x kilometer the distance between S_0 and S, for a is the loss in weight of a one-kilogram stone in a distance of one kilometer.

The procedure is to break a given profile into a series of connected straight lines determine the slope of each section and plot this slope against the distance of its midpoint from the starting place.

The Denwa River was the first one selected to check the foregoing equations. The basic data were taken from the work of Saxena (1993) Circumstances did not permit the calculation of an average line from the slope given there. It was initially assumed for the purpose of this exploratory computation that the profile

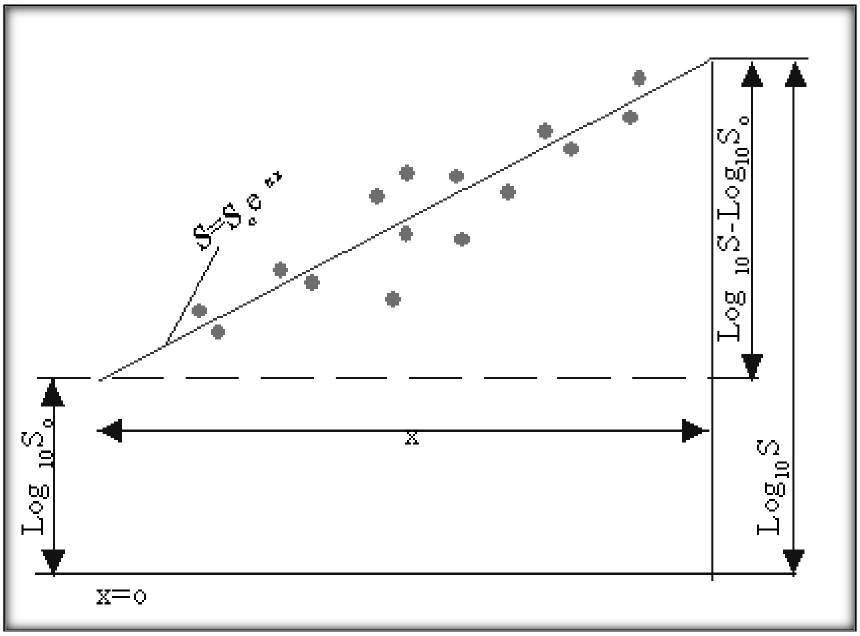


Figure 2. Semi-logarithmic chart of $S = S_0 e^{ax}$

made from the lowest low-water flow would be an adequate approximation of the bed-profile. As $x = 0$ the slopes of the different reaches were plotted against the midpoint of the reach Fig.3.The calculations extended to Denwa Khadd and Denwa Darshan. The points for the main stretch of the river reveal an unmistakable trend though not necessarily a clearly defined straight line. But in view of the complexity of the problem it must be admitted that the results in Figure 3 are really gratifying for too much should not be expected in these initial efforts. The evaluation of the overall wear-coefficient from the straight line put in by eye in Fig 3 is

$$a = \log_{10}(S/S_0)/0.434x = \log_{10}[(0.403/0.130)]/0.434(700) = 0.00162 \; kilogram \; per \; kilogram \; per \; kilometer, \quad (12)$$

A check was made by using the low water profile of the Denwa but no appreciable change in the magnitude of the coefficient resulted. Only the points for the main stretch of the river were used in this determination. The points below Denwa Darshan where excluded because the profile shows changes in structure and stream flow extends up Denwa Darshan; while the Turning point were also not considered as the river-slopes in this reach are known not to conform to Denwa River experience as Fig 3 shows strikingly. The co-efficient thus computed is really not representative of abrasion except in a very loose way; in fact, “abrasion” might be a misleading misnomer in this case. But it can be regarded as a co-efficient of weight decrease that is derived from the actual profile without any implication of the process by which the decrease occurs. In this light the coefficient might be considered an overall characteristic representative of the morphologic behavior of river profiles.

The value of $0.00162 \; kilometer^{-1}$ corresponds to a very tough material slightly less wear-resistant than cost iron. Computed from actual profile-data this coefficient includes the effects of the solids loads of the tributaries. Its smallness may be due to the fineness of the bed-load material and to travel in suspension. A remarkable practical check of this value of 0.00162 has been made by the prominent river engineer, Geraed H. Matthes. He found years ago that pebbles from the Mississippi river near Memphis would scratch the steel blade of his pocket-knife and quite recently, that a piece of gravel would scratch a razor-blade with ordinary hand pressure.

The value of $0.00293 \; kilometer^{-1}$ or kilogram per kilogram per kilometer is obtained for the wear co-efficient for the Bainganga River from the slope of the straight line put in by eye in Fig 4. This would be a material as resistant as a mineral somewhere between porphyry and silicious shale. Which author believes corresponds well with Bainganga material. This means that Denwa River gravel is about twice as tough as Bainganga gravel a fact borne out by practical observation.

A slope distance chart, like Fig 3 or 4, the Denwa River from origin to the boundary line of the Pachmarhis Geological formation demonstrated a similar trend one yielded a value of the wear coefficient very close to that for quartz-which is what would be expected for the material of the Denwa River.

It was found that the slope of the surface of the alluvial bed of the Denwa River confirms to $S = S_0 e^{ax}$ very closely. No attempt was made to compute the wear coefficient

The conclusion is that the Denwa rivers and its sand bank offers corroborative evidence of the validity of the rational equation of $S = S_0 e^{ax}$ for the slopes of river: First, because the plot of the profile-data on a semi-logarithmic chart exhibits an overall linear trend well within the limits of accuracy of such morphologic data; and second because the wear- coefficient derived from the profile agrees astonishingly well with the nature of the bed-load or bed-material of the rivers.

Another form of the profile-equation

A variation of the bed-profile equation in terms of elevation and horizontal distance can be derived from $S = S_0 e^{ax}$. Let z_0 be the elevation at $x = 0$ where the slope is S_0 and let z be the elevation at any point x where the slope is S Fig 1 If dz is the change in elevation in a very short horizontal distance dx. then the slope $S = dz/dx$ and equation (7) becomes

$$S = dz/dx = S_0 e^{ax}, \quad (13)$$

$$dz = S_0 e^{ax} dx, \quad (14)$$

which can be integrated to

$$z - z_0 = (S_0/a)\,(e^{ax} - 1), \quad (15)$$

If the elevation is taken through the origin $z_0 = 0$ and equation (15) can be simplified to

$$z = (S_0/a)(e^{ax} - 1), \quad (16)$$

It must be remembered that equations (15) and (16) are merely derivations from $S = S_0 e \; ax$ and not independent formulas. But they are more directly applicable to profiles and therefore more interesting to the river morphologist.

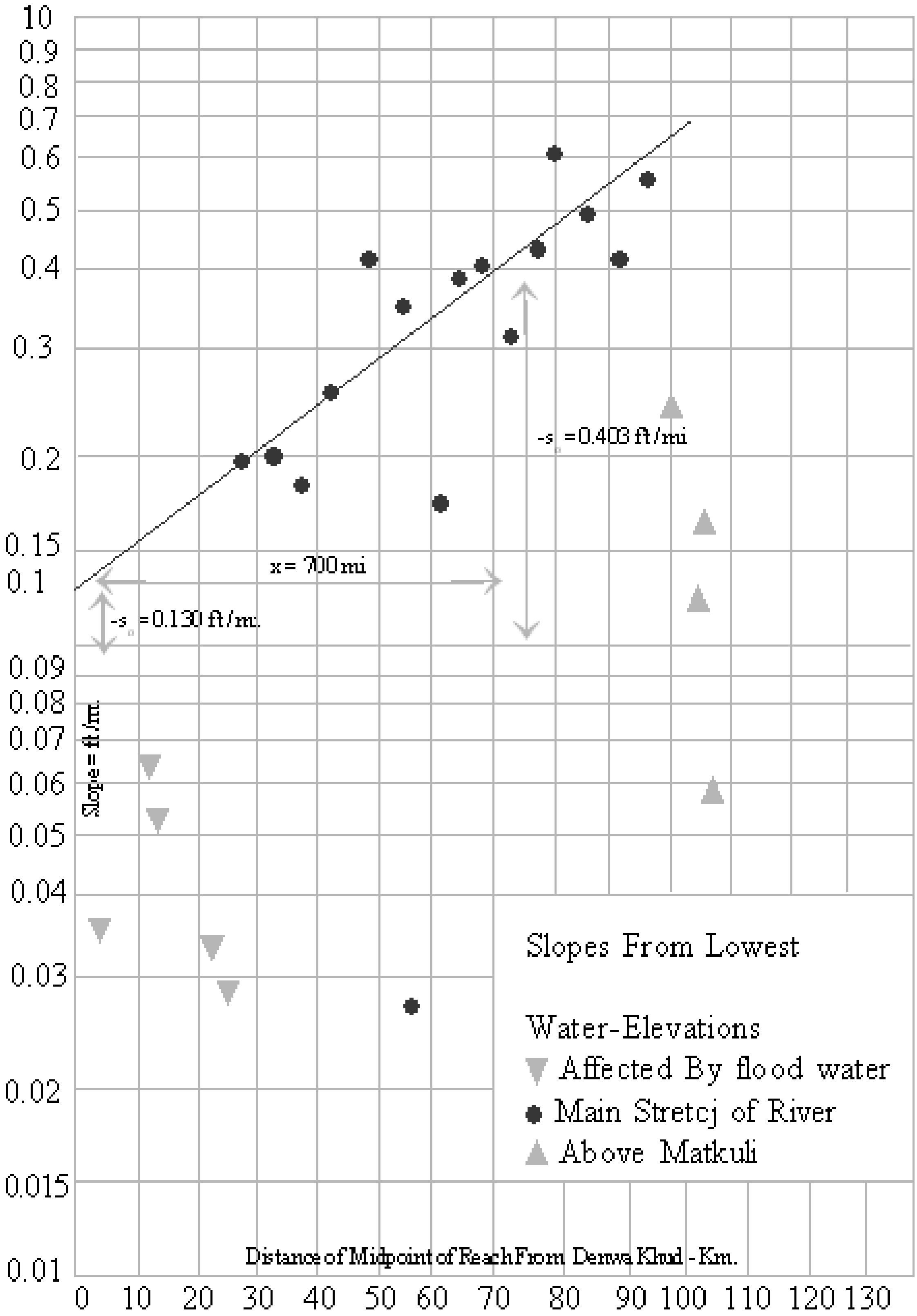


Figure 3 - Variation of low-water slope along Denwa River. Determination of abrasion-coefficient from $S = S_0 e^{ax}$

This different from or equation is not suited to an easy check of the basic principles, though a profile can be computed if the wear-coefficient, a is known. However, Gandolfo tried equation

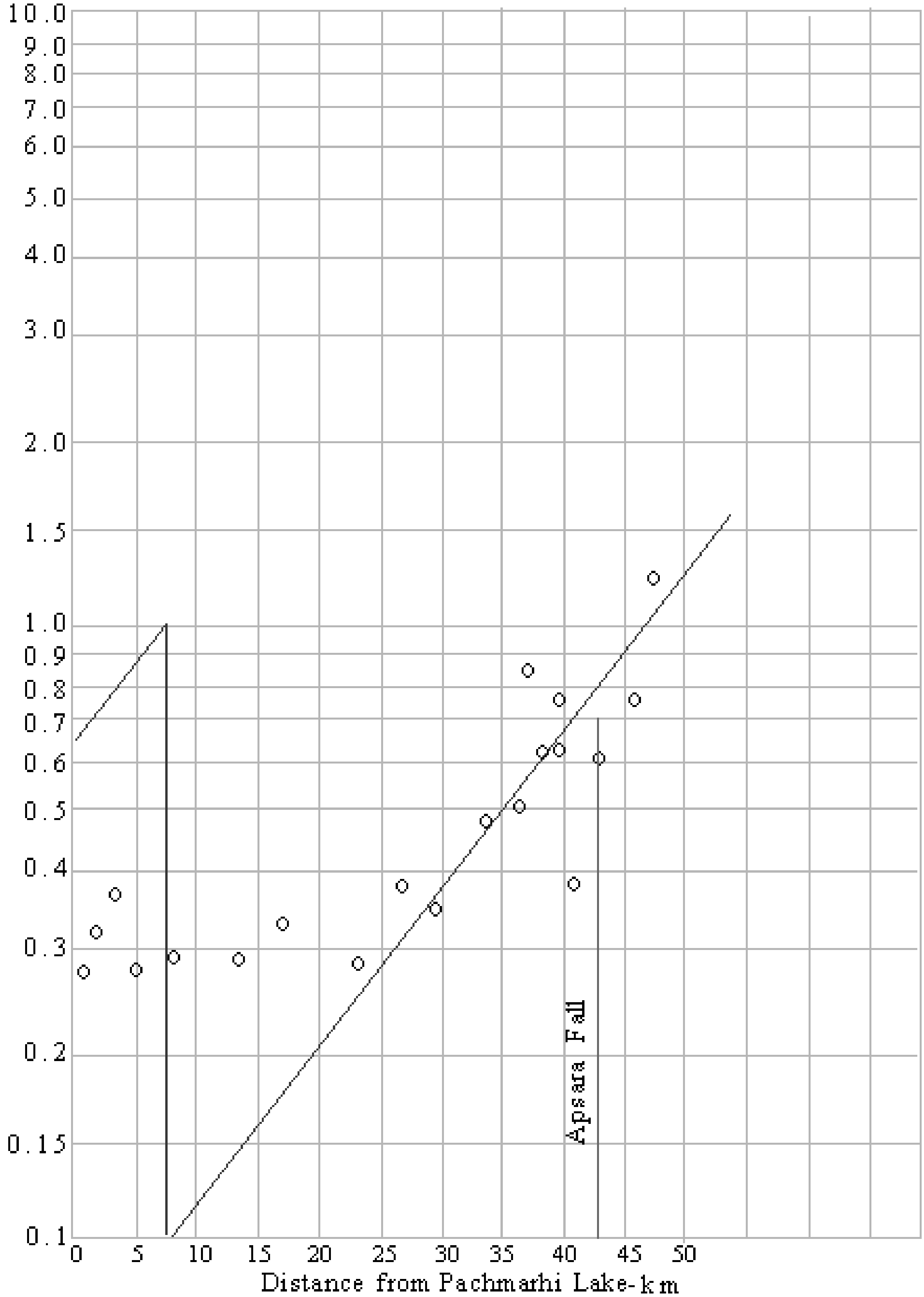


Figure 4. Variation of low-water slope along the Banganga River.

(15) on the San Juan River in Argentina and found agreement of practical adequacy between actual and computed values of the river-profile in an 11.5 kilometer stretch of the river for which comparisons could be made. The coefficient was computed from the mechanical composition curves of the bed-material in a manner

similar to the suggested by Schoklitsch which space requirements prevent from explaining here. Gandolfo's investigation is therefore another verification of the general method.

The semi-logarithmic slope-distance plots (Fig.3 and 4) not being common in practice are not very illuminating and the physical significance is not easily visualized. Furthermore, the scattering of the points on these diagrams might induce doubt and the fitting of a straight line to the data by inspection might be questioned. For this reason the elevation distance equations (15) or (16) are quite reassuring in the diagrams they yield for they show that the straight-line approximation and the resulting value of the wear-coefficient give good elevation-profiles. The gradient of the line on the slope-distance chart cannot be changed too much and still be sensibly acceptable; but the small possible shifts of the line would not alter the computed elevation profile appreciably and the calculated profile is well within reasonable limits of accuracy. To illustrate this, the profiles of the Denwa River will be determined from equation (16).

For rivers, all the variables in equation (16) are easily determinable except the wear- coefficient a. As explained elsewhere (1, 3, 9, 10), it is not a constant for a given rock or bed load material; It increases with the one-fourth power of the particle-velocity, is proportional to the diameter of the material over which the particle rolls, and depends on the pebble shape. That a should vary for different parts of a river is evident, the reduction in weight per kilogram per kilometer decreasing downstream in general. This difficulty is circumvented for river study purposes by computing an overall coefficient for the entire stretch of river under consideration; the shorter the reach, the more representative the coefficient. Yet as will be seen presently, this method yields useful results even for a 1,000 kilometer continuous stretch of river.

Where it is not practicable to work from profile-data, a value of the wear-coefficient can be estimated for the type of mineral in the river from experimentally determined values.

The choice of S_0, the slope at the starting point, $x = 0$, is slightly troublesome. It is best to determine S_0 from the semi- logarithmic chart (Fig.2, 3 and 4). The intersection with $x = 0$ of the straight line through the plotted points yields a good value of S_0 for the purpose.

With $a = 0.00162\ kilometer^{-1}$ from equation (12) and $S_0 = 0.130$ meter per kilometer from Fig 3, equation (16) gives for the elevation profile of the Denwa River

$$Z = (0.130/0.00162)\ (e^{0.00162x} - 1) = 80.2(e^{0.00162x} - 1),` \qquad (17)$$

Values of e^{ax} can be secured from any handbook. The profile given by equation (17) was computed and fitted by inspection to the low-water profile (Fig.4). The calculations were made with $x = 0$ at Denwa Darshan which is affected by tectonic movement. The profile in reference shows that the effect extends up to Denwa khadd. The best visual fit of equation (17) was found to be the portion of the calculated curve upward.

A closer fit might have been obtained by least-square methods. The maximum deviation of the computed profiles from the Denwa low-water line below Denwa Darshan Point is only about six feet. The rather marked divergence above Denwa Darshan is due to the fact that the portion of the river above it was not utilized in the determination of the wear-coefficient from Fig 3.The satisfactoriness of the Denwa River profiles in Fig.5 in no added proof of the basic tenets of this study. Fig. 5 merely affords a better idea of the closeness of fit, of the utility of the method and of the effect of a variation in the wear-coefficient.

$$Z = 80.2\ [e^{0.00162(x-27)} - 1] \qquad (18)$$

For the Bainganga river, with $a = 0.00293\ kilomter^{-1}$ and $S^0 = 0.05\ feet\ per\ kilometer$ from Fig. 4 the derived profile is given by

$$Z = 21.7\ [e^{0.00293(x+10)} - 1] \qquad (19)$$

Shown in figure 6. At kilometer 16, the Little Nagdwari River enters into the Bainganga River, which may explain the divergence of the two lines above this point. Both Figure 4 and 6 depict strikingly the change in regimen at the Bee Fall.

General remarks on applicability and limitations

The rather good result with the Denwa River profiles must not be judged too optimistically or too enthusiastically. Plots of other streams do not always show the regularity of these examples, but in many cases careful inspection of the stream gives a physical interpretation of seeming discrepancies; Rock outcrops, flashy tributaries, debris-cones, local overloading of the stream capacity for solids transport, etc., may disturb the wear phenomena on which the ideas in this study are assumed. In such instances the profile can sometimes by approximated by a series of lines calculated with locally varying coefficients. Steep headwaters in mountainous regions may not fit the framework since collision more than wear might be the predominating agent.

The Denwa, Sonbhadra, Bainganga and Tawa rivers, along with several rivers in India, and Europe substantiate the premise of this paper, that wear-phenomena offer a rational approach to river-bed-profile morphology. The inference is that the bed- load plays a very important role in forming the river- channel. Fig 7

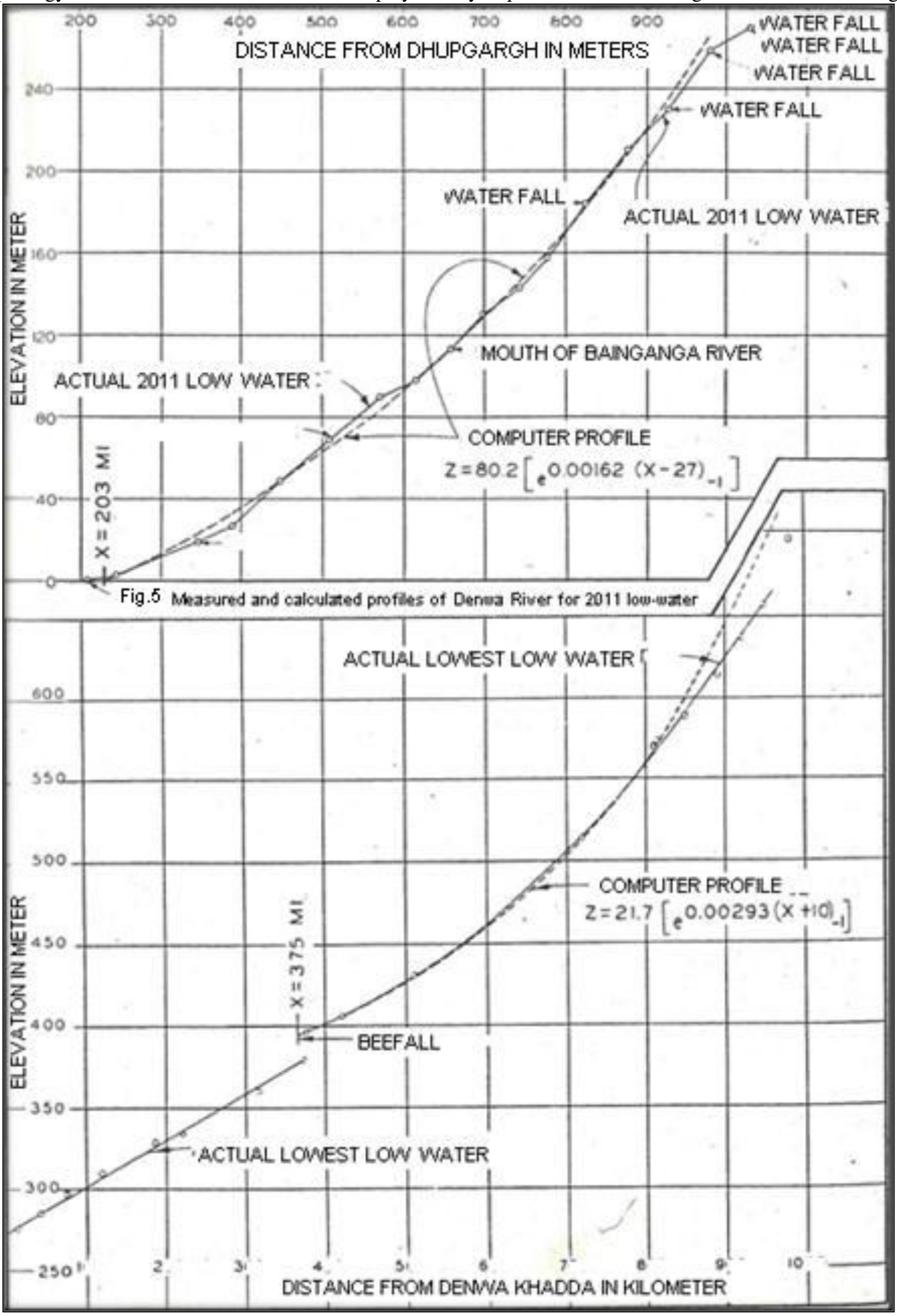


Figure 6. The changes in *b* regimen at the Bee Fall.

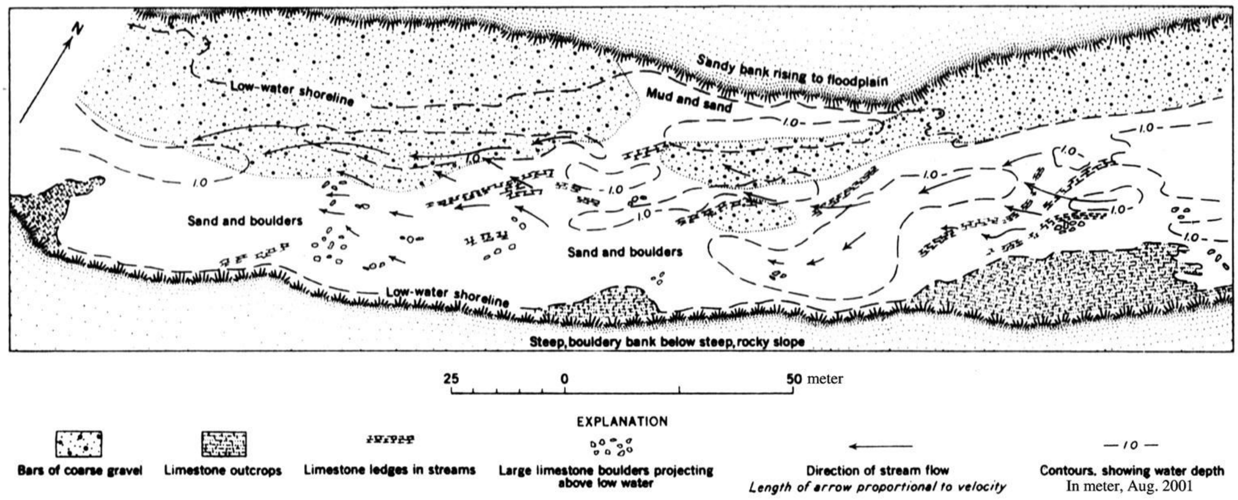


Figure: 7. showing the character of the stream bed. In this reach, the Denwa river flows from east to west at a sharp bend. The caution is indicated, the rational instrument offered here for the analysis of river profiles gives a practical means for expressing the vertical shape of a river-bed so that the morphologic effect of man-made measures can be studied.

Conclusions

The Denwa, Bainganga and Nagduari rivers, in Pachmarhi and an alluvial fan substantiate the premise of this paper, that near-phenomena offer a rational approach to river-bed-profile morphology. The Inference is that the bed-load plays a very important role in forming the river-channel. The caution is indicated, the rational Instrument offered here for the analysis of river- profiles gives a practical means for expressing the vertical shape of a river-bed so that the morphologic effect of man-made measures can be studied. The usefulness of $S = S^0 e^{ax}$ and $Z = (S_0/a)e^{ax} - 1)$ lies in the fact that they formulate the equilibrium- profile, that which will prevail ultimately or after complete morphologic development. Hence, if the regime of a river is altered by regulation, a practical prediction of the final form can be ventured, with more rational basis for the estimate than has been the case hitherto.

A fanciful possibility is the establishment of a relation between an overall wear-coefficient like "a" and the watershed geology that would permit fluvial or physiographic predictions in a manner akin to the use of flood-formulas, certainly an accuracy, equal to the dubious one of this common hydrologic tools should be attainable.

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